CE-ITI and the Institute of Computer Science

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Czech Academy of Sciences

In names . . .

One FTE every year distributed among:

- Five researchers:
  - Petr Cintula
  - Zuzana Haniková
  - Rostislav Horčík 2012–2017
  - Petr Savický
  - Jiří Šíma
- Five postdocs:
  - Paolo Baldi 2018
  - Stefano Bonzio 2017
  - Tommaso Moraschini 2016–2017
  - Igor Sedlár 2016–2017
  - Tomoyuki Suzuki 2013–2014
- One PhD student:
  - Adam Přenosil 2015–2018

. . . and numbers

- 20 journal publications
- 7 proceedings papers
- 3 book chapters
- 1 defended PhD
- 1 award of Czech Academy of Sceinces
- 1 major international conference
- 7 workshops and one-day seminars
- 2 advanced semestral courses
- 8 related grant projects

ICS in CE-ITI's three Key Activities

(as specified in CE-ITI project proposal)

KA1 Conducting research of the highest quality,
addressing major challenges and open problems,
and initiating new lines of research

KA2 Educating a new generation of researchers,
actively searching for new talents

KA3 Coordinating and fostering international cooperation,
strengthening the worldwide standing of Czech
computer science and mathematics

KA1: Research

We have focused on two Work Packages

- WP6 Computational complexity
- WP8 Substructural logics in computer science

more about this later ...

KA2: Education and new talents

- We thought two semestral courses at Charles University
- Adam Přenosil defended excellent PhD thesis and currently works as postdoc at Vanderbilt University
- We have attracted five postdocs from abroad
- One of them, Tommaso Moraschini, received award of Czech Academy of Sciences and of Hlávka foundation for excellence in research
- Other one, Igor Sedlár, become principal investigator of his own junior GACR grant
- We have organized
  - three annual Prague meetings of Logicians
  - Summer School PhD in Logic
  - Summer School affiliated to conference Topology, Algebra, and Categories in Logic

KA3: International cooperation

- We have organized major conference Topology, Algebra, and Categories in Logic for cca 150 participants
- We organized workshop ManyVal 2013 for cca 50 participants
- We organized a series of one-day seminars
  - Prague seminar: The Future of Mathematical Fuzzy Logic 2016
  - Prague seminar on Non-Classical Mathematics 2015
  - Prague Seminar on Substructural Logics 2014
  - 2nd Prague Symposium on Semilinear Logics 2013
- We are partners at extensive H2020 MSCA-RISE project Syntax meets semantics
- We received joint project of Czech and Austrian Science foundations

Let us get back to KA1

WP 8

Substructural logics (SLs)

Classical logic has (among others) the following structural rules

$$\frac{\Gamma, A, B \Rightarrow C}{\Gamma, B, A \Rightarrow C}$$ order of premises doesn't matter

$$\frac{\Gamma, A, A \Rightarrow C}{\Gamma, A \Rightarrow C}$$ multiplicity of premises doesn't matter

$$\frac{\Gamma \Rightarrow C}{\Gamma, A \Rightarrow C}$$ new premisses can be added

These rules have been challenged for various reasons by logicians,
philosophers, linguists, and computer scientists

Substructural logics are a family of non-classical logics
without some of these rules

Substructural logics (SLs)

Two research lines:

- Mathematical study of particular SLs and of their general theory
- Applications of particular SLs for various CS problems
  - separation logic for reasoning about pointer programs
  - Lambek calculus for study of categorial grammars
  - Linear logic for formalizing resource-sensitive tasks

As members of department of Theoretical Computer Science working
on the project called Institute for Theoretical Computer Science ...

...we have produced a quantity of high quality research in the first line

A list of notable outcomes 1 — core theory

We have contributed to the core theory of Substructural logics:

Petr Cintula and Rosta Horčík wrote an extensive paper on the general theory of non-associative SLs

Zuzana Haniková and Petr Savický gave a comprehensive study of SAT problem for certain SLs

Rosta Horčík explored of connection between decidability issues for SLs and the theory of regular languages

Petr Cintula studied of the Skolemization and Herbrandization in SLs, a first step in automated deduction for these logics

A list of notable outcomes 2 — abstract approaches

We have also contributed to abstract study of non-classical logics:

Petr Cintula and Tomasso Moraschini generalized the Tarskian notion of consequence relation as a relation between finite multisets of formulas; thus creating a framework better suited for the abstract study of SLs

Tommaso Moraschini has investigated the complexity of the problem of classifying semantically-presented logics inside the Leibniz hierarchy;[1] in particular he showed that this problem is EXPTIME-complete

[1] The most important classification hierarchy for non-classical logics and a central notion of Abstract Algebraic Logic

A list of notable outcomes 3 — paraconsistent logics

Of particular logics we have mainly focused on paraconsistent logics aiming at more realistic handling of contradictions than provided by the classical explosive approach:[2]

Adam Přenosil has initiated a systematic study of extensions of the prominent Belnap-–Dunn logic which resulted in four papers and his excellent PhD thesis

Igor Sedlár studied paraconsistent dynamic extension of Belnap-–Dunn logic, described its axiomatization and proved its decidability

Igor Sedlár showed that prominent four-valued modal logic BK can be seen as an epistemic logic and derived interesting consequences of this fact

[2] i.e., approach when deriving a formula and its negation allows to derive everything

WP 6

Derandomizing the Logarithmic Space

RL is the class of problems solvable in logarithmic space (& polynomial time) with probabilistic Turing machines with one-sided error (TMs may reject incorrectly)

conjecture: **RL = L ?**

derandomization using a polynomial-time hitting set generator: Construct a hitting set H such that for every algorithm $A(x, r)$ with randomnes r and every input x,

$$Pr_{r\sim U_n}[A(x, r) = 1] \geq \varepsilon \quad \text{implies} \quad (\exists\, r^* \in H)\, A(x, r^*) = 1$$

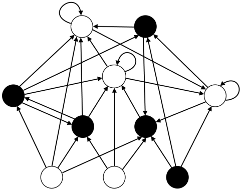
`Šíma, Žák:` a non-trivial (40-page proof) polynomial-time construction of a hitting set for read-once branching programs of width 3 ($\varepsilon > \frac{5}{6}$)

- such a result for polynomial width would imply RL=L
- previous constructions known only for severely restricted oblivious (regular, permutation) read-once BPs of bounded width; the case of width 4 still open
- attracted approx. 25 citations; active research topic (STOC'18, FOCS'18, etc.)

The Energy Complexity of Neural Networks

only about 1% of neurons fire concurrently in the brain due to the limited energy (oxygen) supply

vs.



approx. half of neurons output 1 in parallel in a typical NN model that is designed artificially

Energy Complexity = the maximum number of neurons firing at the same time instant, over all possible computations

Šíma:

- the definition of energy for cyclic (recurrent) NNs
- the energy-time tradeoff in a size-optimal implementation of finite automata
- lower bounds on the energy consumption (communication complexity)

The Computational Power of Neural Networks

depends on the information contents of weight parameters:

1. integer weights: finite automaton (Minsky, 1967)

2. rational weights: Turing machine (Siegelmann, Sontag, 1995)

   polynomial time ≡ complexity class P

   polynomial time & increasing Kolmogorov complexity of real weights ≡
   a proper hierarchy of nonuniform complexity classes between P and P/poly
   (Balcázar, Gavaldà, Siegelmann, 1997)

3. arbitrary real weights: "super-Turing" computation
   (Siegelmann, Sontag, 1994)

   polynomial time ≡ nonuniform complexity class P/poly
   exponential time ≡ any I/O mapping

Between Integer and Rational Weights

rANN is a binary-state neural network with r extra analog neurons

Šíma: refining the analysis of NNs within the Chomsky hierarchy:

rational-weight 3ANNs ≡ TMs ≡ recursively enumerable languages **(Type-0)**

online 1ANNs ⊂ LBA ≡ context-sensitive languages **(Type-1)**

1ANNs ⊄ PDA ≡ context-free languages **(Type-2)**

integer-weight NNs ≡ **"quasi-periodic"** 1ANNs ≡ FA ≡ regular languages **(Type-3)**

Non-Standard Positional Numeral Systems

- a real base (radix) β such that $|\beta| > 1$
- a finite set $A \neq \emptyset$ of real digits

β-expansion of a real number $x \in \mathbb{R}$ using the digits from $a_k \in A$, $k \geq 1$:

$$x = (0\,.\,a_1\,a_2\,a_3\ \ldots)_\beta = \sum_{k=1}^{\infty} a_k \beta^{-k}$$

Examples:

- decimal expansions: $\beta = 10$, $A = \{0, 1, 2, \ldots, 9\}$

  $\frac{3}{4} = (0\,.\,74\,\overline{9})_{10} = 7 \cdot 10^{-1} + 5 \cdot 10^{-2} + 9 \cdot 10^{-3} + 9 \cdot 10^{-4} + \cdots$

  any number has at most 2 decimal expansions, e.g. $(0\,.\,74\,\overline{9})_{10} = (0\,.\,75\,\overline{0})_{10}$

- non-integer base: $\beta = \frac{5}{2}$, $A = \left\{0, \frac{1}{2}, \frac{7}{4}\right\}$

  $\frac{3}{4} = \left(0\,.\,\frac{7}{4}\,\frac{1}{2}\,0\,\overline{\frac{7}{4}\,0}\right)_{\frac{5}{2}} = \frac{7}{4} \cdot \left(\frac{5}{2}\right)^{-1} + \frac{1}{2} \cdot \left(\frac{5}{2}\right)^{-2} + 0 \cdot \left(\frac{5}{2}\right)^{-3} + \frac{7}{4} \cdot \left(\frac{5}{2}\right)^{-4} + \cdots$

  most of the representable numbers has a continuum of distinct β-expansions,

  e.g. $\frac{3}{4} = \left(0\,.\,\overline{\frac{7}{4}\,\frac{1}{2}\,\frac{1}{2}\,\ldots\,\frac{1}{2}\,0}\right)_{\frac{5}{2}}$

Quasi-Periodic β-Expansion

eventually periodic β-expansions:

$$\Big(0\,.\,\underbrace{a_1\,\ldots\,a_{m_1}}_{\text{preperiodic part}}\;\underbrace{\overline{a_{m_1+1}\,\ldots\,a_{m_2}}}_{\text{repetend}}\Big)_\beta \qquad \left(\text{e.g. } \frac{19}{55} = (0\,.\,3\,\overline{45})_{10}\right)$$

eventually quasi-periodic β-expansions:

$$\Big(0\,.\,\underbrace{a_1\ldots a_{m_1}}_{\text{preperiodic part}}\;\underbrace{a_{m_1+1}\ldots a_{m_2}}_{\text{quasi-repetend}}\;\underbrace{a_{m_2+1}\ldots a_{m_3}}_{\text{quasi-repetend}}\;\underbrace{a_{m_3+1}\ldots a_{m_4}}_{\text{quasi-repetend}}\;\ldots\Big)_\beta$$

such that

$$(0\,.\,\overline{a_{m_1+1}\ldots a_{m_2}})_\beta = (0\,.\,\overline{a_{m_2+1}\ldots a_{m_3}})_\beta = (0\,.\,\overline{a_{m_3+1}\ldots a_{m_4}})_\beta = \cdots$$

Example: the plastic $\beta \approx 1.324718$ $(\beta^3 - \beta - 1 = 0)$, $A = \{0, 1\}$

$$1 = (0\,.\,0\;\underbrace{100}\;\underbrace{0\,011\,011\,1}\;\underbrace{0\,011\,1}\;\underbrace{100}\;\ldots)_\beta$$

with quasi-repetends: $(0\,.\,\overline{100})_\beta = (0\,.\,\overline{0(011)^i1})_\beta = \beta$ for every $i \geq 1$

Quasi-Periodic Numbers

$r \in \mathbb{R}$ is a β-quasi-periodic number within A if every β-expansions of r is eventually quasi-periodic

Examples:

- r from the complement of the Cantor set **is** 3-quasi-periodic within $A = \{0, 2\}$ (r has no β-expansion at all)
- $r = \frac{3}{4}$ **is** $\frac{5}{2}$-quasi-periodic within $A = \left\{0\,,\ \frac{1}{2}\,,\ \frac{7}{4}\right\}$
- $r = 1$ **is** β-quasi-periodic within $A = \{0, 1\}$ for the plastic $\beta \approx 1.324718$
- $r \in \mathbb{Q}(\beta)$ **is** β-quasi-periodic within $A \subset \mathbb{Q}(\beta)$ for Pisot β (a real algebraic integer $\beta > 1$ whose all Galois conjugates $\beta' \in \mathbb{C}$ satisfy $|\beta'| < 1$)
- $r = \frac{40}{57} = (0\,.\,0\,\overline{011})_{\frac{3}{2}}$ **is not** $\frac{3}{2}$-quasi-periodic within $A = \{0, 1\}$ (greedy $\frac{3}{2}$-expansion of $\frac{40}{57} = (0\,.\,100000001\ldots)_{\frac{3}{2}}$ is not event. quasi-periodic)

Eventually Quasi-Periodic β-Expansions

for $a = a_1\, a_2\, a_3 \ldots \in A^\omega$, denote the set of tail values,

$$R(a) = \left\{ (0\,.\,a_{k+1}\, a_{k+2}\, a_{k+3} \ldots)_\beta \,\middle|\, k \geq 0 \right\}$$

Šíma, Savický:

- If $R(a)$ is finite, then $(0\,.\,a_1\, a_2\, a_3 \ldots)_\beta$ is eventually quasi-periodic.
- Let β be a real algebraic number whose all conjugates β' meet $|\beta'| \neq 1$. Then

  $R(a)$ is finite **iff** $(0\,.\,a_1\, a_2\, a_3 \ldots)_\beta$ is eventually quasi-periodic.
- Let β be a real algebraic number whose conjugate β' meets $|\beta'| = 1$.

  Then there exist integer digits $A \subset \mathbb{Z}$ and a quasi-periodic β-expansion $(0\,.\,a_1\, a_2\, a_3 \ldots)_\beta = 0$ with infinite $R(a)$.

  (solves an important open problem in algebraic number theory)