Properties of the weighted and robust implicitly weighted correlation coefficients

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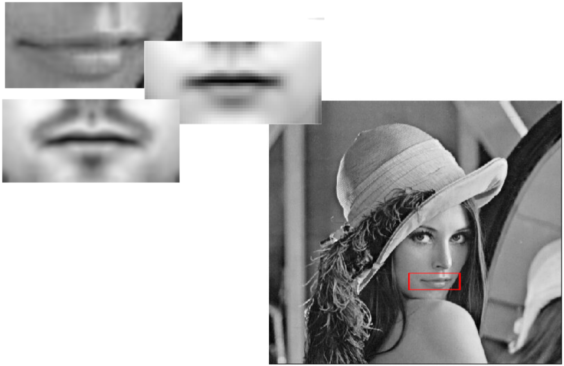
Introduction

Template matching

- important part of computer vision algorithms
- searching for object or part of object in the image
- often performed in CNN feature space



Template matching

Sparse template

- templates can be sparse
- more efficient calculation

Template matching

Sub-tasks

- find optimal template/centroid
- find set of candidate regions
- evaluate the similarity between region and centroid

Similarity measures

- correlation coefficient
- Pearson product-moment correlation coefficient
  - similarity of two vectors
  - large variety of use cases
  - used in context of deep learning

Correlation coefficients

- r Pearson product-moment correlation coefficient

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

- r_w weighted version of Pearson correlation coefficient
  - in some applications pairs of values are assigned **weights** (according measurement errors, position in image, etc.)

$$r = \frac{\sum_{i=1}^{n} w_i(x_i - \bar{x}_w)(y_i - \bar{y}_w)}{\sqrt{\sum_{i=1}^{n} w_i(x_i - \bar{x}_w)^2}\sqrt{\sum_{i=1}^{n} w_i(y_i - \bar{y}_w)^2}}$$

Energetic demands

Use on small devices

- need for low energy demands
- need for low memory demands
- various approximate computations are used

Upper and lower bounds for r_w

- one of results of our paper
- while using approximate versions of target vectors
- enables template matching on small devices

Modified image

Effect of modifying the image

- asymmetric illumination
- rotation

Result

- object localisation based on template matching with r_w is robust only to very small rotations of the image
- it is also vulnerable to illumination
- this may be argument for moving template matching to feature space (derived by CNN)

LWS-based robust correlation coefficient

LWS-based robust correlation coefficient

- variant of weighted correlation coefficient
- more robust
- based on Least Weighted Squares regression

  `Kalina, J.: Robust coefficients of correlation or spatial autocorrelation based on implicit weighting. Journal of the Korean Statistical Society 51, 1247-1267 (2022)`

LWS estimator

- linear regression
- weight function - non-increasing continuous function

$$\psi : [0,1] \to [0,1], \text{where } \psi(0) = 1, \psi(1) = 0$$

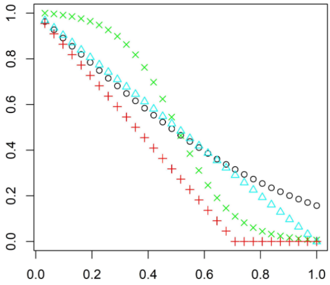
LWS-based robust correlation coefficient

LWS estimator

- LWS estimator is defined as

$$\arg\min_{b \in \mathbb{R}^d} \sum_{i=1}^{n} w_i u_{(i)}^2(b)$$

- residuals $u_{(i)}^2(b)$ are arranged in ascending order



LWS-based robust correlation coefficient

Definition

- regression task

$$y_i = \beta_0 + \beta_1 x_i + e_i$$

- r_{LWS}:

$$r_{LWS}(x, y) = r_{LWS}(x, y, \tilde{w})$$

- $\tilde{w}$ optimal permutation of weights given by LWS

Properties

- highly robust alternative to r especially with respect to outliers

Hypothesis test based on r_{LWS}

Hypothesis about correlation coefficient

- Null hypothesis H_0: There is no significant correlation between the variables ($\rho = 0$)
- Alternate hypothesis H_1: There is a significant correlation between the variables ($\rho \neq 0$).

Test statistic T

$$T_{LWS} = \frac{r_{LWS}(x,y)}{\sqrt{1 - r_{LWS}^2(x,y)}}\sqrt{n-2}$$

- asymptotically random variable with normal distribution

Hypothesis test based on r_{LWS}

Test result

$$H_0 \text{ is rejected } \iff |T_{LWS}| \geq z_{1-\alpha/2}$$

Test based on r_{LWS} and the Fisher transform

- test statistic

$$Z_{LWS} = \frac{1}{2} \log \left(\frac{1 + r_{LWS}}{1 - r_{LWS}} \right)$$

- test result

$$H_0 \text{ is rejected} \iff Z_{LWS}/SD(Z_{LWS}) \geq z_{1-\alpha/2}$$

Conclusion

- correlation coefficient and its weighted versions used in many applications, including template matching
- several properties derived including using approximate centroids (templates)
- tools for hypothesis testing derived

Thank you!