On Robust Training of Regression Neural Networks

Jan Kalina, Petra Vidnerová

The Czech Academy of Sciences, Institute of Computer Science, Prague

- Neural networks are commonly used for models with a large number of variables (or even for functional data)
- Here: **nonlinear regression** of unknown form
- Model

$$Y_i = \varphi(X_i) + e_i$$

for $i = 1, \ldots, n$, where $e_1, \ldots, e_n$ are mutually independent and $X_i \in \mathbb{R}^p$

- Standard training of common types of neural networks minimize the sum of squared residuals (mean squared error – MSE)
- Thus, they may be heavily influenced by contamination (**outliers**)

$\Longrightarrow$ **need for robust neural networks training**

- Multi-layer perceptron neural networks (MLP)
- Radial basis function networks (RBF)

Neural networks with a robust loss function

LTS-MLP, LTS-RBF

Neural networks with the loss function of the least trimmed squares (LTS) defined for a fixed h ($n/2 \leq h < n$):

$$\arg\min_{\hat{\theta} \in \mathbb{R}^N} \left\{ \sum_{i=1}^{h} u_{(i)}^2(\hat{\theta}) + \lambda \sum_{j=1}^{Q} |\hat{\theta}_i| \right\}$$

LTA-MLP, LTA-RBF

Neural networks with the loss function of least trimmed absolute error (LTA):

$$\arg\min_{\hat{\theta} \in \mathbb{R}^N} \left\{ \sum_{i=1}^{h} |u(\hat{\theta})|_{(i)} + \lambda \sum_{j=1}^{Q} |\hat{\theta}_i| \right\}$$

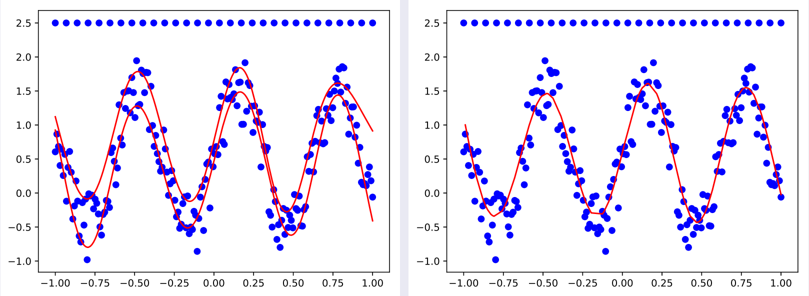
- $\hat{\theta} \in \mathbb{R}^Q$ = estimate of network parameters θ
- $u_i(\hat{\theta})$ = residuals for $i = 1, \ldots, n$
- $u_{(1)}^2(\hat{\theta}) \leq \cdots \leq u_{(n)}^2(\hat{\theta})$, $|u(\hat{\theta})|_{(1)} \leq \cdots \leq |u(\hat{\theta})|_{(n)}$
- Regularization parameter $\lambda > 0$ (found by cross validation)

Quantile regression neural networks: QMLP(τ), QRBF(τ)

- modified loss function
- approximation of quantiles

Interquantile neural networks (IQ-MLP, IQ-RBF)

Simple illustrative example



Summary

Studied models

- MLP, RBF
- back-MLP, back-RBF - robust version based on backward subsample selection
- LTA-MLP, LTA-RBF - least trimmed absolute error version
- LTS-MLP, LTS-RBF - least trimmed squared error version
- IQ-MLP, IQ-RBF - inter quantile networks

Conclusion

- Several novel methods turn out to be suitable for various types of contamination (also under no contamination)
- The interquantile approach seems to be the most promising (also for heteroscedastic models)
- Future work: Hard trimming may be replaced by implicit weights, metalearning may propose suitable weights