Proper multi-type display calculi for classical and intuitionistic inquisitive logic

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Joint work with Sabine Frittella, Fan Yang
and Alessandra Palmigiano

Outline

Inquisitive logic (Ciardelli, Groenendijk and Roelofsen 2009)

Assertions

Questions

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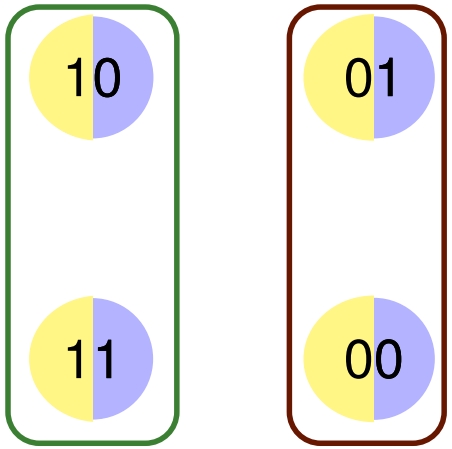

An *information state* (or *team*):
a set of valuations

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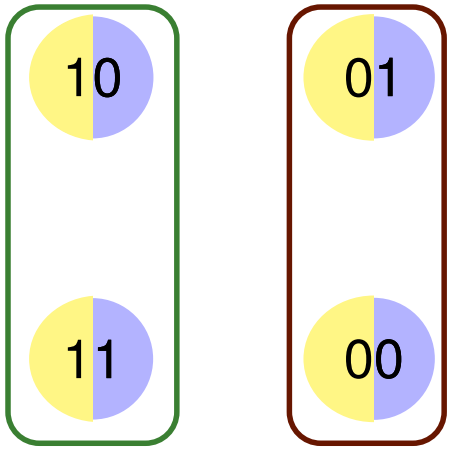

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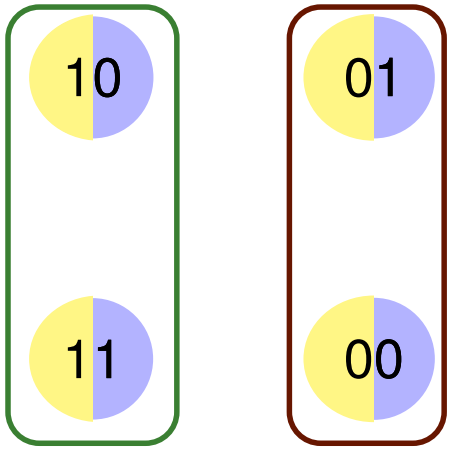

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Team Semantics (Hodges 1997)

Applied to *Dependence logic* (Väänänen 2007)

Inquisitive logic (InqL)

Syntax

$$\phi ::= p \mid \bot \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \to \phi \quad (\neg\phi ::= \phi \to \bot)$$

- $S \models p$ iff for all $v \in S$, $v(p) = 1$
- $S \models \bot$ iff $S = \emptyset$
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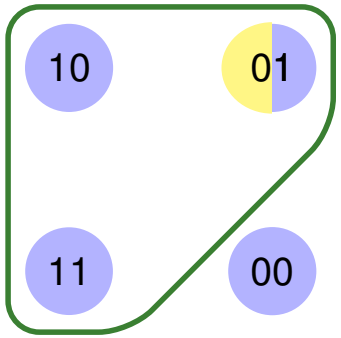

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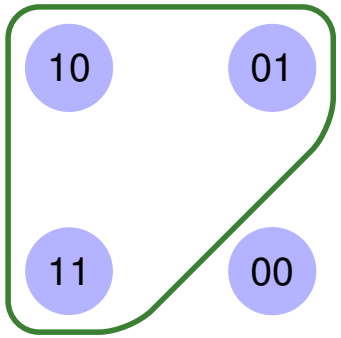

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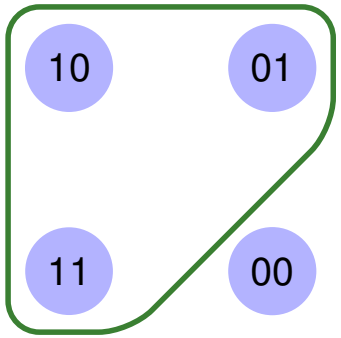

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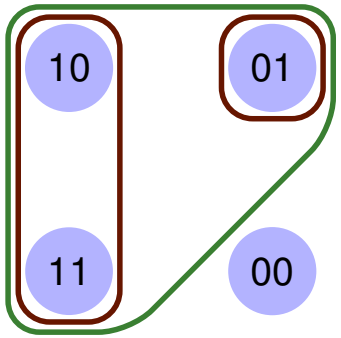

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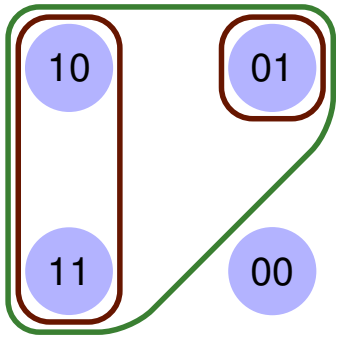

(Downward Closure) For every formula ϕ of InqL,

$$T \subseteq S \models \phi \Longrightarrow T \models \phi$$

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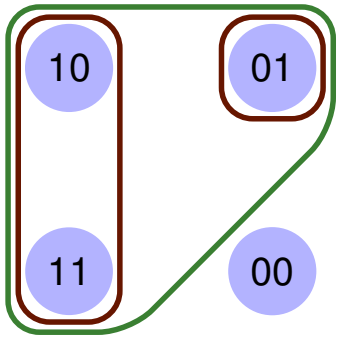

Def. A formula ϕ is said to be flat iff for all teams S,

$$S \models \phi \iff \forall v \in S,\; v(\phi) = 1.$$

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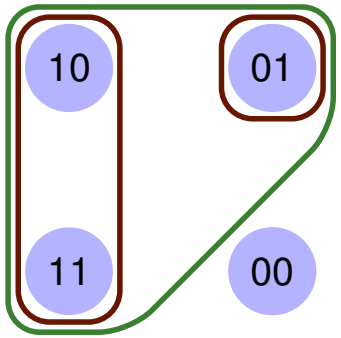

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Fact: A formula α is flat iff it is equivalent to some $\vee$-free formula.

iff $\neg\neg\alpha \equiv \alpha$

Theorem (Ciardelli, Roelofsen, 2009)

The following Hilbert-style system of InqL *is sound and complete:*

Axioms:

1. **IPC** *axiom schemata*
2. *Kreisel-Putnam axiom schemata:*

$$(\neg\phi \to (\psi \vee \chi)) \to (\neg\phi \to \psi) \vee (\neg\phi \to \chi)$$

3. $\neg\neg p \to p$

Rule:

Modus Ponens

- InqL $= KP \oplus \neg\neg p \to p$
- InqL is NOT closed under uniform substitution.

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3. $\neg\neg\alpha \to \alpha$ *for any flat formula* α

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A multi-type inquisitive logic

Flat Type

General Type

Fix a set V of propositional variables.

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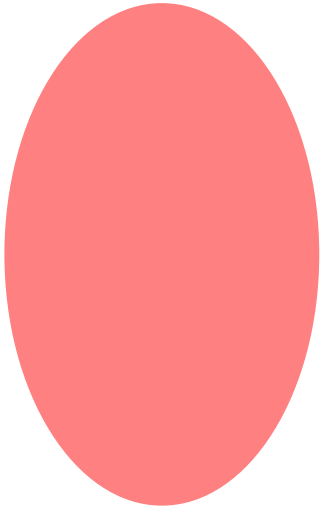
$$\mathbb{B} = (\wp(2^V), \cap, \cup, (\cdot)^c, \emptyset, 2^V)$$



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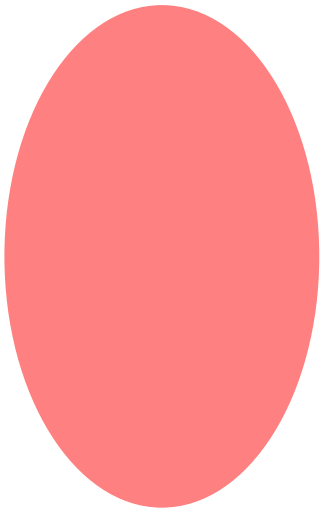

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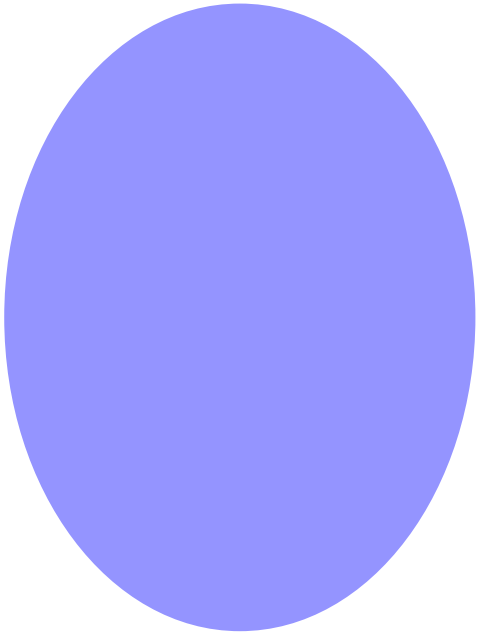

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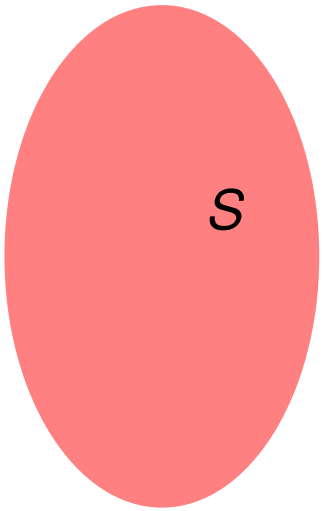

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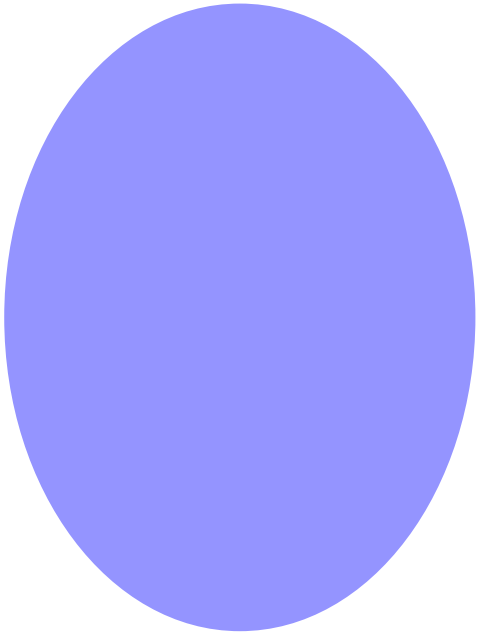

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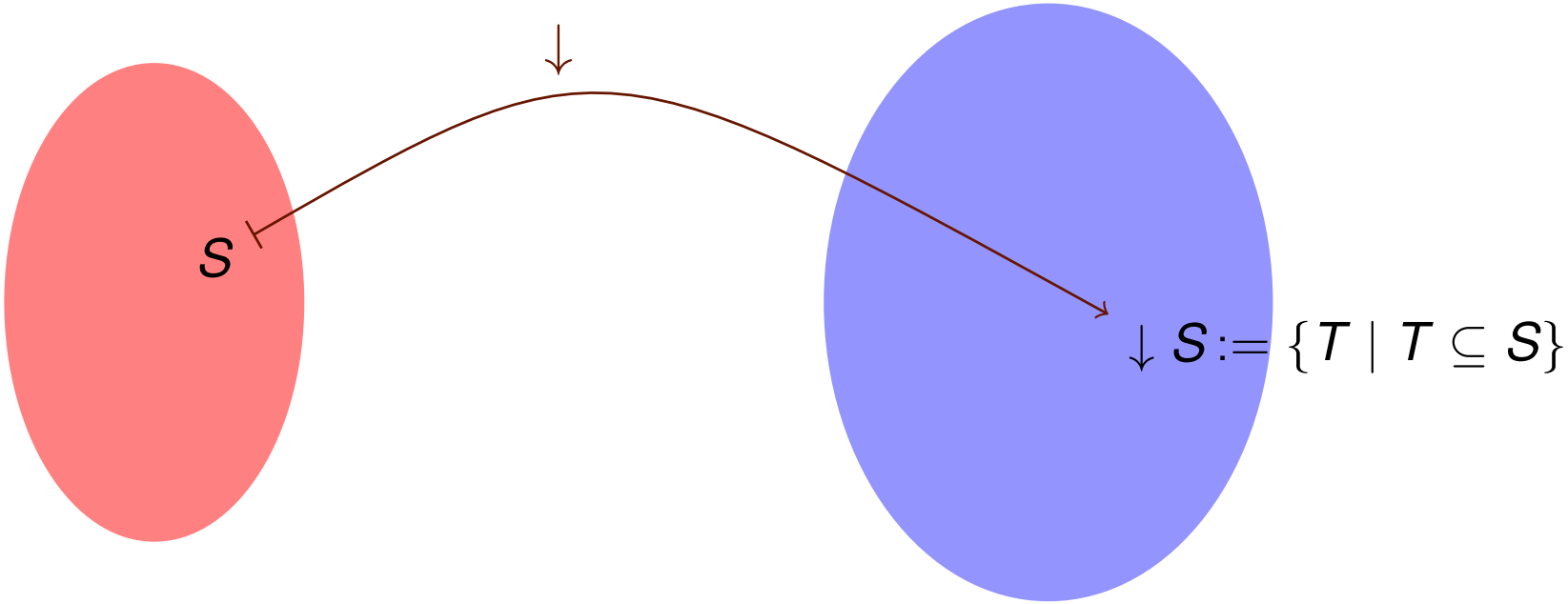

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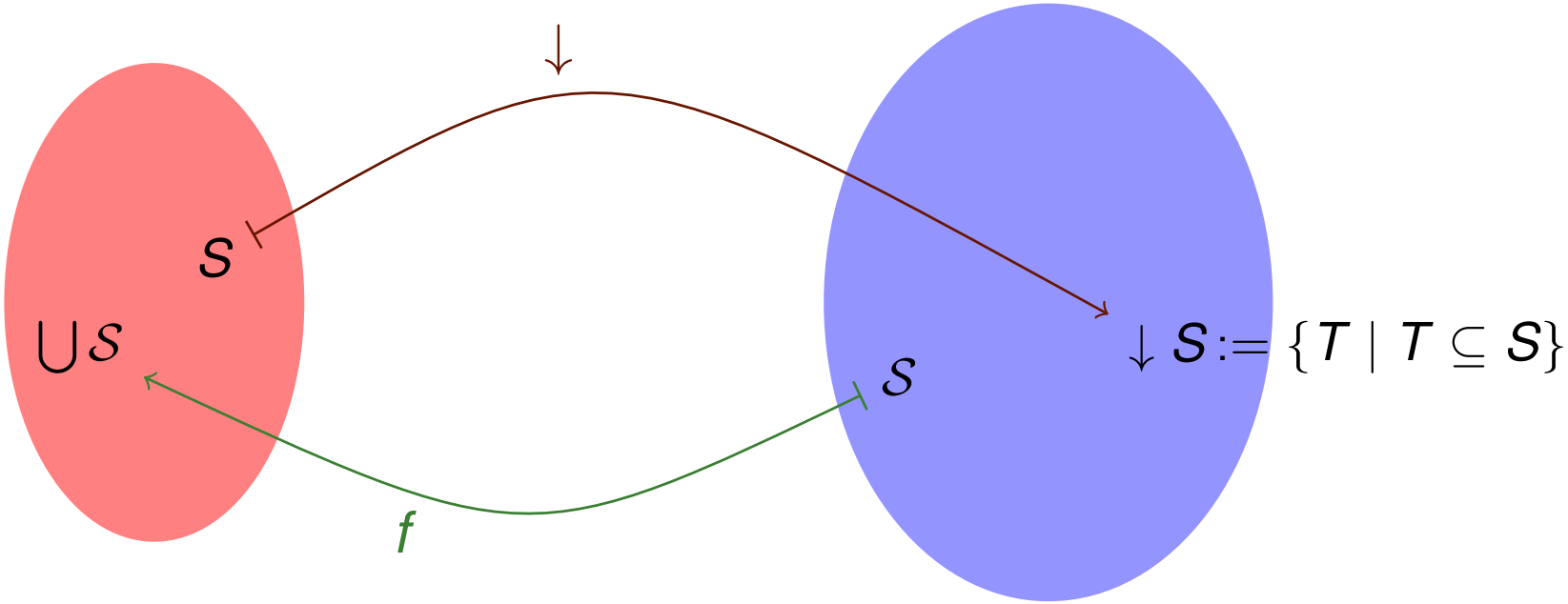

$$f \dashv \downarrow$$

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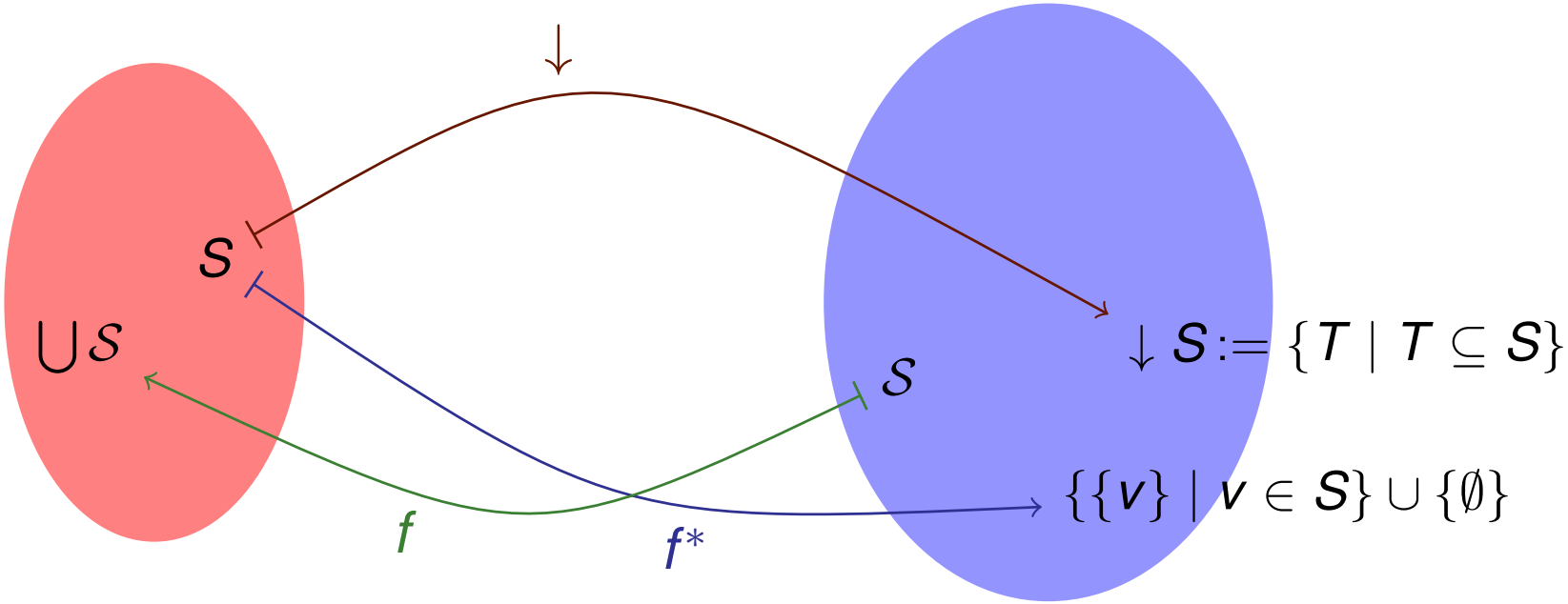

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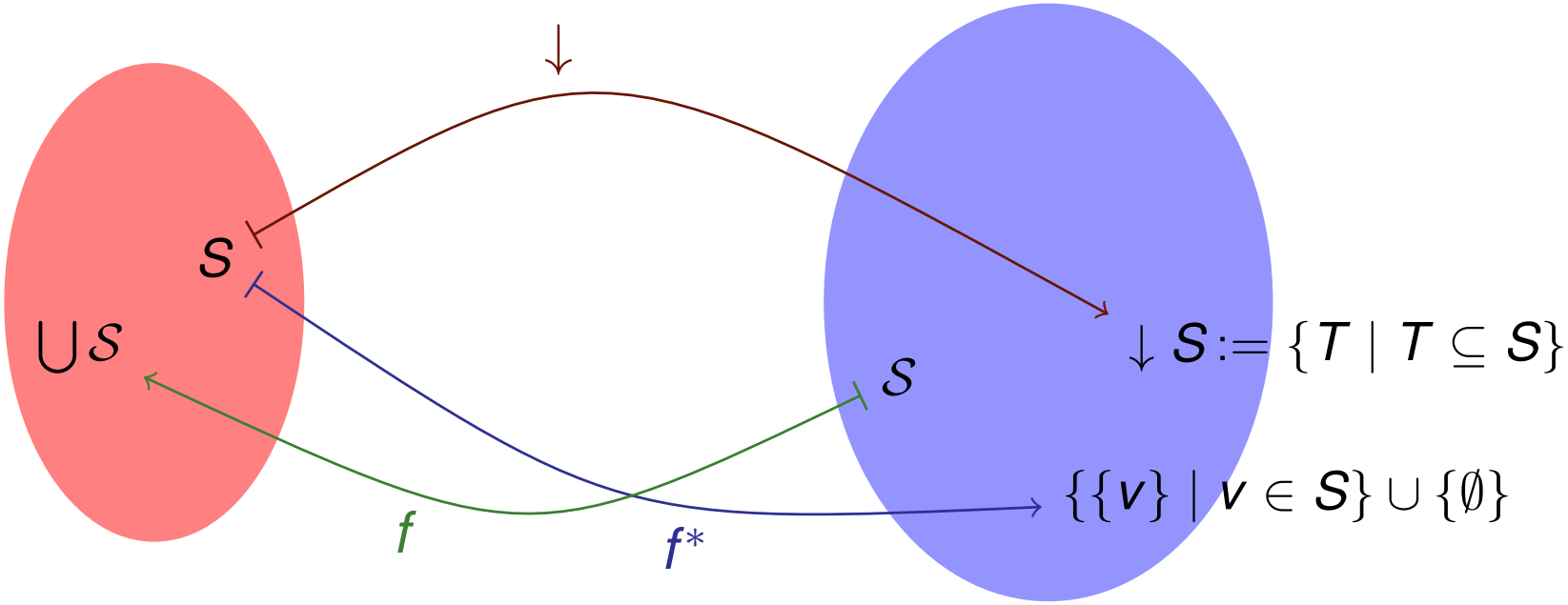

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Multi-type inquisitive logic:

$$\text{Flat} \ni \alpha ::= p \mid 0 \mid \alpha \sqcap \alpha \mid \alpha \multimap \alpha$$

$$\text{General} \ni A ::= \downarrow\alpha \mid A \wedge A \mid A \vee A \mid A \to A$$

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  - (A1) **CPC** axiom schemata for Flat-formulas
  - (A2) **IPC** axiom schemata for General-formulas
  - (A3) $(\downarrow\alpha \rightarrow (A \vee B)) \rightarrow (\downarrow\alpha \rightarrow A) \vee (\downarrow\alpha \rightarrow B)$
  - (A4) $\neg\neg\downarrow\alpha \rightarrow \downarrow\alpha$
- Rule:

  Modus Ponens for formulas of both types

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$(f^* \dashv f \dashv {\downarrow})$

Intermezzo on proof theory

Canonical cut elimination, 1/2

Definition

A sequent $x \vdash y$ is ***type-uniform*** if x and y are of the same type.

A (cut) rule is ***strongly type-uniform*** if its premises and conclusion are of the same type.

Theorem (Canonical cut elimination)

If a calculus satisfies the properties below, then it enjoys cut elimination.

Canonical cut elimination, 2/2

1. structures can disappear, formulas are **forever**;
2. **tree-traceable** formula-occurrences, via suitably defined congruence:
   - same shape, same position, same type, non-proliferation;
3. **principal = displayed**
4. rules are closed under **uniform substitution** of congruent parameters within each type;
5. **reduction strategy** exists when cut formulas are both principal.

Specific to multi-type setting:

6. **type-uniformity** of derivable sequents;
7. **strongly uniform cuts** in each/some type(s).

A multi-type sequent calculus for inquisitive logic

Structural and operational languages

Flat

$$\alpha ::= p \mid 0 \mid \alpha \sqcap \alpha \mid \alpha \multimap \alpha$$

$$\Gamma ::= \alpha \mid \Phi \mid \Gamma, \Gamma \mid \Gamma \sqsupset \Gamma \mid \mathrm{F}X$$

General

$$A ::= \downarrow\alpha \mid A \wedge A \mid A \vee A \mid A \to A$$

$$X ::= A \mid \Downarrow\Gamma \mid \mathrm{F}^*\Gamma \mid X ; X \mid X > X$$

Structural and operational languages

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$$A ::= \downarrow\alpha \mid A \wedge A \mid A \vee A \mid A \rightarrow A$$

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Interpretation of structural connectives

Flat connectives:

Structural symbols	Φ		$,$		$\sqsupset$	
Operational symbols	(1)	0	$\sqcap$	$(\sqcup)$	$(\mapsto)$	$\rightarrowtriangle$

General connectives:

Structural symbols	$;$		$>$	
Operational symbols	$\wedge$	$\vee$	$(\rightarrowtail)$	$\rightarrow$

Multi-type connectives: $(f^* \dashv f \dashv \downarrow)$

Structural symbols	F^*		F		$\Downarrow$	
Operational symbols	(f^*)		(f)	(f)		$\downarrow$

Cut rules

$$\frac{\Gamma \vdash \alpha \qquad \alpha \vdash \Delta}{\Gamma \vdash \Delta}\ \mathit{Cut} \qquad\qquad \frac{X \vdash A \qquad A \vdash Y}{X \vdash Y}\ \mathit{Cut}$$

Structural rules

Flat type:

$$\mathit{Id}\ \frac{}{p \vdash p} \qquad \frac{\Pi \vdash \Gamma \sqsupset (\Delta\,,\Sigma)}{\Pi \vdash (\Gamma \sqsupset \Delta)\,,\Sigma}\ \mathrm{CG} \qquad \frac{\Pi \vdash (\Gamma \sqsupset \Delta)\,,\Sigma}{\Pi \vdash \Gamma \sqsupset (\Delta\,,\Sigma)}\ \mathrm{IG}$$

Interaction between the two types:

$$\frac{\Gamma \vdash \Delta}{\mathrm{F}^*\Gamma \vdash \Downarrow\Delta}\ \mathrm{bal} \qquad\qquad \frac{X \vdash Y}{\mathrm{F}X \vdash \mathrm{F}Y}\ \mathrm{f\ mon}$$

$$\frac{\mathrm{F}^*\Gamma \vdash \Delta}{\Gamma \vdash \mathrm{F}\Delta}\ \mathrm{f\ adj} \qquad\qquad \frac{\mathrm{F}X \vdash \Gamma}{X \vdash \Downarrow\Gamma}\ \mathrm{d\ adj} \qquad\qquad \frac{X \vdash \Downarrow\mathrm{F}Y}{X \vdash Y}\ \mathrm{d\text{-}f\ elim}$$

$$\frac{X \vdash \Downarrow(\Gamma \sqsupset \Delta)}{X \vdash \mathrm{F}^*\Gamma > \Downarrow\Delta}\ \mathrm{d\ dis} \qquad\qquad \frac{\mathrm{F}X\,,\mathrm{F}Y \vdash Z}{\mathrm{F}(X\,;Y) \vdash Z}\ \mathrm{f\ dis}$$

$$\frac{X \vdash \mathrm{F}^*\Gamma > (Y\,;Z) \qquad X \vdash \mathrm{F}^*\Gamma > (Y\,;Z)}{X \vdash (\mathrm{F}^*\Gamma > Y)\,;(\mathrm{F}^*\Gamma > Z)}\ \mathrm{KP}$$

Completeness 1/2

$$
\dfrac{\dfrac{\dfrac{\dfrac{\alpha \vdash \alpha}{\mathrm{F}^*\alpha \vdash \Downarrow\alpha}\ \text{bal}}{\mathrm{F}^*\alpha \vdash \downarrow\alpha} \qquad \dfrac{B \vdash B \qquad C \vdash C}{B \vee C \vdash B\,;C}}{\downarrow\alpha \to (B \vee C) \vdash \mathrm{F}^*\alpha > (B\,;C)} \qquad \dfrac{\dfrac{\dfrac{\alpha \vdash \alpha}{\mathrm{F}^*\alpha \vdash \Downarrow\alpha}\ \text{bal}}{\mathrm{F}^*\alpha \vdash \downarrow\alpha} \qquad \dfrac{B \vdash B \qquad C \vdash C}{B \vee C \vdash B\,;C}}{\downarrow\alpha \to (B \vee C) \vdash \mathrm{F}^*\alpha > (B\,;C)}}{\dfrac{\downarrow\alpha \to (B \vee C) \vdash (\mathrm{F}^*\alpha > B)\,;(\mathrm{F}^*\alpha > C)}{\downarrow\alpha \to (B \vee C) \vdash (\downarrow\alpha \to B) \vee (\downarrow\alpha \to C)}\ \text{f adj, d-f elim}}\ \text{KP}
$$

Completeness 2/2

$$
\begin{prooftree}
\AxiomC{$\alpha \vdash \alpha$}
\RightLabel{}
\UnaryInfC{$\alpha \vdash 0\,,\alpha$}
\RightLabel{}
\UnaryInfC{$\alpha\,,\Phi \vdash 0\,,\alpha$}
\RightLabel{}
\UnaryInfC{$\Phi \vdash \alpha \sqsupset (0\,,\alpha)$}
\RightLabel{CG}
\UnaryInfC{$\Phi \vdash (\alpha \sqsupset 0)\,,\alpha$}
\RightLabel{}
\UnaryInfC{$\Phi \vdash \alpha\,,(\alpha \sqsupset 0)$}
\RightLabel{}
\UnaryInfC{$\alpha \sqsupset \Phi \vdash \alpha \sqsupset 0$}
\RightLabel{bal}
\UnaryInfC{$\mathrm{F}^*(\alpha \sqsupset \Phi) \vdash \Downarrow(\alpha \sqsupset 0)$}
\RightLabel{d dis, f adj, d-f elim}
\UnaryInfC{$\Downarrow(\alpha \sqsupset \Phi) \vdash \neg\downarrow\alpha$}
\AxiomC{$0 \vdash \Phi$}
\RightLabel{}
\UnaryInfC{$\downarrow 0 \vdash \Downarrow\Phi$}
\RightLabel{}
\BinaryInfC{$\neg\downarrow\alpha \to \downarrow 0 \vdash \Downarrow(\alpha \sqsupset \Phi) > \Downarrow\Phi$}
\RightLabel{def}
\UnaryInfC{$\neg\neg\downarrow\alpha \vdash \Downarrow(\alpha \sqsupset \Phi) > \Downarrow\Phi$}
\RightLabel{d dis}
\UnaryInfC{$\neg\neg\downarrow\alpha \vdash \Downarrow((\alpha \sqsupset \Phi) \sqsupset \Phi)$}
\RightLabel{d adj}
\UnaryInfC{$\mathrm{F}\neg\neg\downarrow\alpha \vdash (\alpha \sqsupset \Phi) \sqsupset \Phi$}
\RightLabel{IG}
\UnaryInfC{$\mathrm{F}\neg\neg\downarrow\alpha \vdash \alpha$}
\RightLabel{d adj}
\UnaryInfC{$\neg\neg\downarrow\alpha \vdash \Downarrow\alpha$}
\RightLabel{}
\UnaryInfC{$\neg\neg\downarrow\alpha \vdash \downarrow\alpha$}
\end{prooftree}
$$

Future work: intuitionistic inquisitive logic

(Ciardelli, Iemhoff and Yang 2017)

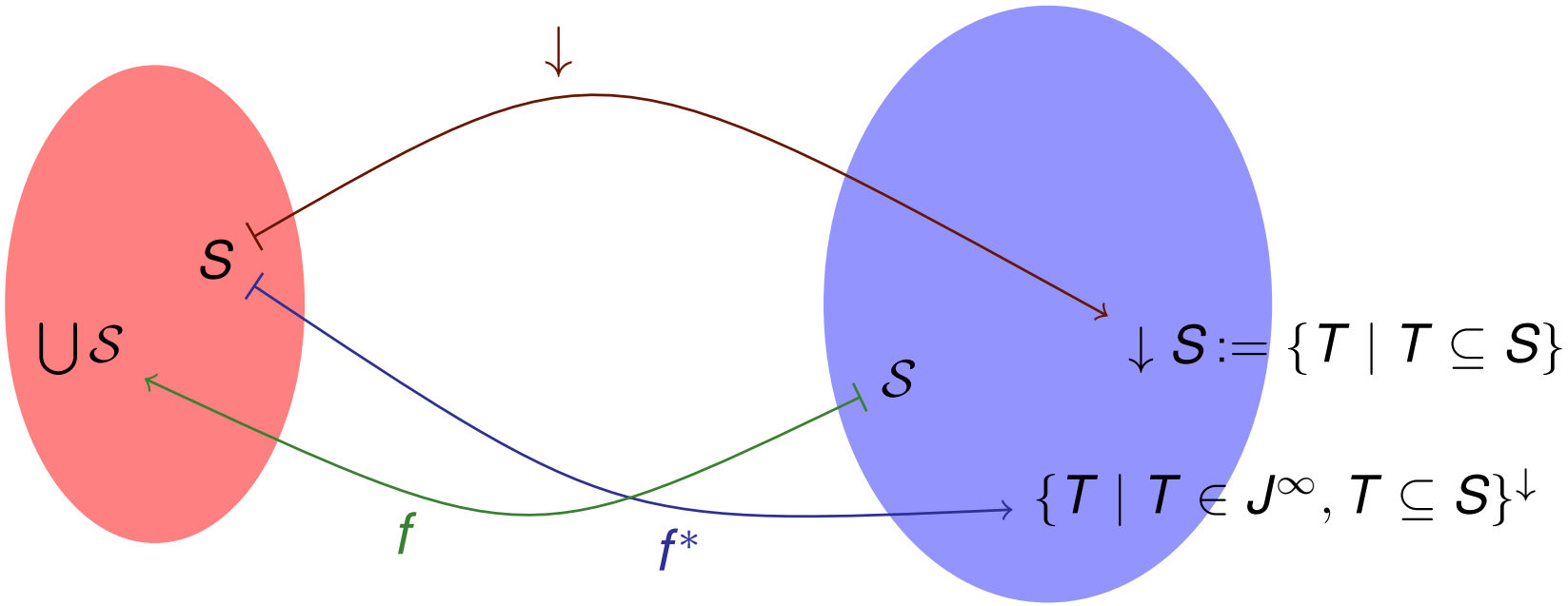
Fix an set V of propositional variables.

Flat Type

$$\mathbb{B}' = (\wp^{\downarrow}(2^V), \cap, \cup, \Rightarrow, \emptyset, 2^V)$$

General Type

$$\mathbb{A}' = (\wp^{\downarrow}(\mathbb{B}'), \cap, \cup, \Rightarrow, \emptyset, \wp(2^V))$$



$$f^* \dashv f \dashv \downarrow$$