A General Tableau Method for Deciding Description Logics, Modal Logics and Related First-Order Fragments

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joint work with
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Outline

Increasing Demand for Reasoning Tools

- Description logics form a basis for web ontology languages, OWL DL and OWL 1.1
- Modal and dynamic logics are useful in multi-agent reasoning
- Metric logics are intended to be helpful in classification problems
- Fuzzy logics . . . , etc

Reasoning tools are of increased demand.

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Reasoning Tools

- Tableau provers: FACT++, RACERPRO, PELLET, DLP, ...
  are highly optimised but not generic
- Resolution provers: MSPASS, VAMPIRE, ...
  are difficult to tune to decide a particular logic (a first-order fragment)
- Generic interactive platforms: ISABELLE, COQ, ...
- Tableau prover engineering platforms: LWB, TWB, LOTREC, ...
- Other: KAON2, ...

Answer to the increased demand for decision procedures for logical systems from existing automated reasoning tools is not sufficient.

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Our Approach
Tableau Prover Synthesis

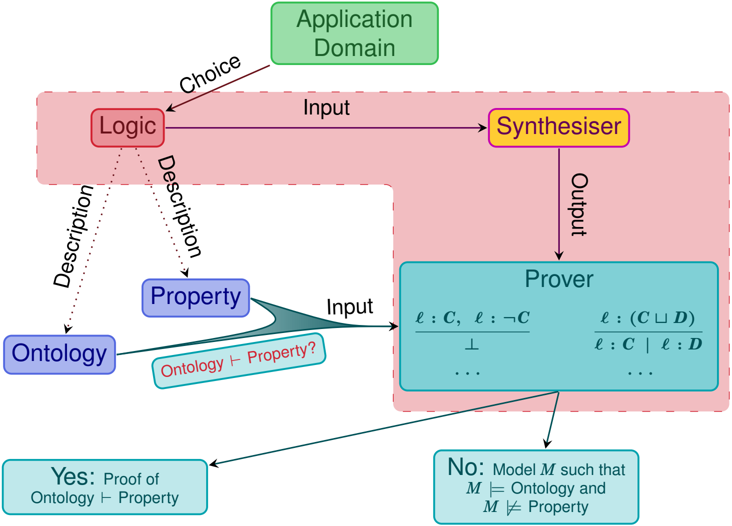

Tableau Termination Problem

- How to ensure termination of a tableau algorithm?
- An appropriate blocking mechanism is needed, e.g.:
  - subset or equality blocking,
  - dynamic or static blocking,
  - successor or anywhere blocking,
  - combinations of the above.
- Problem: How to
  - define a general blocking mechanism which unifies all the standard ones and
  - describe a class of logics for which the general blocking mechanism ensures termination of corresponding tableau algorithms?

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- Absence of general decision procedures in automated reasoning for tableaux and instantiation-based methods.
- Absence of a theoretical foundations for generic platforms in which tableau decision procedures can be built in a uniform way for different logics and different applications.
- The work is based on observation that proofs of termination of tableau algorithms and proofs of the effective finite model property by the filtration argument are very similar.

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Syntax and Semantics

individual

Concepts: $C, D \stackrel{\text{def}}{=} p \mid \neg C \mid C \sqcup D \mid \exists R.C \mid \{\ell\} \mid \ell : C$

Roles: $R, R_i \stackrel{\text{def}}{=} r \mid \rho_0(R_1, \ldots, R_{\mu_0}) \mid \rho_1(R_1, \ldots, R_{\mu_1}) \mid \ldots$

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Interpretation (model): $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ satisfying

$$p^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \qquad r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \qquad \ell^{\mathcal{I}} \in \Delta^{\mathcal{I}}$$

$$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} \qquad (C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$$

$$(\exists R.C)^{\mathcal{I}} = \{x \mid \exists y \in C^{\mathcal{I}}\ (x, y) \in R^{\mathcal{I}}\} \qquad (\{\ell\})^{\mathcal{I}} = \{\ell^{\mathcal{I}}\}$$

$$(\ell : C)^{\mathcal{I}} = \begin{cases} \Delta^{\mathcal{I}}, & \text{if } \ell^{\mathcal{I}} \in C^{\mathcal{I}}, \\ \varnothing, & \text{otherwise, and} \end{cases}$$

additional semantic conditions for $\rho_0, \rho_1, \ldots$

.

Example: $\mathcal{SO}$ — Logic with Transitive Roles

- Language extended by transitive role constants $s \in$ Trans.
- For every $s \in$ Trans and a model $\mathcal{I}$, the interpretation of $s^{\mathcal{I}}$ is a transitive relation on $\mathcal{I}$:
  $(x, y), (y, z) \in s^{\mathcal{I}}$ implies $(x, z) \in s^{\mathcal{I}}$ for all $x, y, z \in \Delta^{\mathcal{I}}$.

Example: $\mathcal{ALBO}$ — Logic with Boolean Role Operators

- Extra operators on roles: role inverse R^{-1}, role complement $\neg R$, and role union $R \sqcup S$.
- Interpretations of the operators:

$$(\neg R)^{\mathcal{I}} \stackrel{\text{def}}{=} (\Delta \times \Delta) \setminus R^{\mathcal{I}}$$
$$(R \sqcup S)^{\mathcal{I}} \stackrel{\text{def}}{=} R^{\mathcal{I}} \cup S^{\mathcal{I}}$$
$$(R^{-1})^{\mathcal{I}} \stackrel{\text{def}}{=} (R^{\mathcal{I}})^{-1} = \{(x,y) \mid (y,x) \in R^{\mathcal{I}}\}$$

Properties

- $\mathcal{ALBO}$ is out of the mainstream DLs.
- $\mathcal{ALBO}$ subsumes two variable fragment of first-order logic.
- $\mathcal{ALBO}$ is decidable by resolution.
- Satisfiability problem for $\mathcal{ALBO}$ is NExpTime-complete.
- Very expressive: universal modality and Boolean combinations of role inclusions $R \sqsubseteq S$, concept inclusions $C \sqsubseteq D$, concept assertions $\ell : C$, role assertions $(\ell, \ell') : D$, etc are expressible in $\mathcal{ALBO}$.

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Closure operator

- sub is a monotone operator on sets of expressions.
- $\Sigma \subseteq \text{sub}(\Sigma)$ for every Σ.
- sub is *finite* iff $\text{sub}(\Sigma)$ is finite whenever Σ is finite.
- A finite sub can be replaced by an equivalent notion of a well-founded ordering on expressions.
- Σ is sub-*closed*, or a *signature* iff $\Sigma = \text{sub}(\Sigma)$.
- Usually, there is a lot of flexibility in choice of sub.

Example

- sub for $\mathcal{SO}$ and $\mathcal{ALBO}$ can be chosen as the subexpression operator, i.e. $\text{sub}(\Sigma)$ is a set of all subexpressions of expressions in Σ.
- sub for *PDL* includes more expressions.

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Filtration

- $\mathcal{I}$ is a model
- $\sim$ is an equivalence relation on $\Delta^{\mathcal{I}}$
- $[x] \stackrel{\text{def}}{=} \{y \in \Delta^{\mathcal{I}} \mid x \sim y\}$
- *Filtration* of $\mathcal{I}$ is a structure $\overline{\mathcal{I}} = (\Delta^{\overline{\mathcal{I}}}, .^{\overline{\mathcal{I}}})$ such that
  - $\Delta^{\overline{\mathcal{I}}} = \{[x] \mid x \in \Delta^{\mathcal{I}}\}$,
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- *L admits finite filtration* iff for every finite L-signature Σ and every L-model $\mathcal{I}$ of the signature Σ there exists an equivalence relation $\sim$ on $\mathcal{I}$ such that there is a $\sim$-filtration $\overline{\mathcal{I}}$ of $\mathcal{I}$ which is a finite L-model of the signature Σ.

Theorem

Let L be a logic and sub *be a finite expression closure operator. If L admits finite filtration then L has the effective finite model property.*

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- L *admits finite filtration* iff for every finite L-signature Σ and every L-model $\mathcal{I}$ of the signature Σ there exists an equivalence relation $\sim$ on $\mathcal{I}$ such that there is a $\sim$-filtration $\overline{\mathcal{I}}$ of $\mathcal{I}$ which is a finite L-model of the signature Σ.

Theorem

Let L be a logic and sub *be a finite expression closure operator. If L admits finite filtration then L has the effective finite model property.*

$\mathcal{SO}$ Filtration

- Given an $\mathcal{SO}$-model $\mathcal{I}$ and a signature Σ, let

$$\tau^{\Sigma}(x) \stackrel{\text{def}}{=} \{C \in \Sigma \mid x \in C^{\mathcal{I}}\}.$$

- The equivalence $\sim$ defined by

$$x \sim y \stackrel{\text{def}}{\Longleftrightarrow} \tau^{\Sigma}(x) = \tau^{\Sigma}(y)$$

  for every $x, y \in \Delta^{\mathcal{I}}$.
- An interpretation of every role r in the $\sim$-filtration $\overline{\mathcal{I}}$ of $\mathcal{I}$ is defined by

$$r^{\overline{\mathcal{I}}} \stackrel{\text{def}}{=} \{([x],[y]) \mid y \in C^{\mathcal{I}} \text{ implies } x \in (\exists r.C)^{\mathcal{I}} \text{ for every } \exists r.C \in \Sigma\}.$$

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  - $R^{\overline{\mathcal{I}}} \stackrel{\text{def}}{=} \{(\lfloor x \rfloor, \lfloor y \rfloor) \mid \exists x' \simeq x \exists y' \simeq y \, (x', y') \in R^{\mathcal{I}}\}$.
  - It is finite but, in general, does not produce an $\mathcal{ALBO}$-model: the property $(\neg R)^{\mathcal{I}} \subseteq (\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}) \setminus R^{\mathcal{I}}$ is affected.
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Example of a Nice $\mathcal{ALBO}$ Filtration

$$x \cong y \overset{\text{def}}{\Longleftrightarrow} \tau^{\Sigma}(x) = \tau^{\Sigma}(y) \text{ and }$$
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It is not finite!

Counterexample

Example of a Nice $\mathcal{ALBO}$ Filtration

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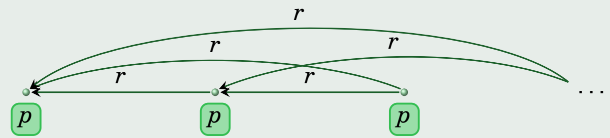

Conflict Elimination

- Introduced by Gargov, Passy, and Tinchev for *BML*.
- Works for *BML* and $\mathcal{ALB}$ but, in general, fails if individuals are in the language.
- *Quasi-model*: $\mathcal{I}$ where (possibly) $(\neg R)^{\mathcal{I}} \not\subseteq (\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}) \setminus R^{\mathcal{I}}$.
- If $\mathcal{ALB}$-quasi-model $\mathcal{I}$ is finite and Σ is a finite signature then there are
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Theorem

If a $\mathcal{ALB}$-concept C is satisfiable in a quasi-model then it is satisfiable in a finite model.

Corollary

- *$\mathcal{ALB}$ is complete with respect to the class of all $\mathcal{ALB}$-quasi-models.*
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A Finite Nice $\mathcal{ALBO}$ Filtration

- Take an $\mathcal{ALBO}$-concept C and an $\mathcal{ALBO}$-model $\mathcal{I}$ satisfying C.
- Replace all singleton subconcepts $\{\ell\}$ in C by fresh propositional symbols p_ℓ. Let C' be the result of the replacement and $\Sigma \stackrel{\text{def}}{=} \text{sub}(C')$.
- Make an $\mathcal{ALB}$-model $\mathcal{I}'$ from $\mathcal{I}$ by making interpretation $p_\ell^{\mathcal{I}'} \stackrel{\text{def}}{=} \{\ell\}^{\mathcal{I}}$. Clearly, $\mathcal{I}'$ satisfies C'.
- Obtain a finite $\mathcal{ALB}$-quasi-model $\underline{\mathcal{I}}$ satisfying C using the standard filtration on $\mathcal{I}'$.
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Theorem

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$$\tau^{\Sigma}(u,z) = \tau^{\Sigma}(u,z) \text{ and } \tau^{\Sigma}(z,u) = \tau^{\Sigma}(z,u).$$

- Replace p_ℓ back for $\{\ell\}$ in C', Σ, and $\mathcal{I}'$ and apply the defined nice filtration to the original $\mathcal{I}$.

Theorem

- $\mathcal{ALBO}$ is complete with respect to the class of all $\mathcal{ALBO}$-quasi-models.
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A Finite Nice $\mathcal{ALBO}$ Filtration

- Take an $\mathcal{ALBO}$-concept C and an $\mathcal{ALBO}$-model $\mathcal{I}$ satisfying C.
- Replace all singleton subconcepts $\{\ell\}$ in C by fresh propositional symbols p_ℓ. Let C' be the result of the replacement and $\Sigma \stackrel{\text{def}}{=} \text{sub}(C')$.
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Tableau Calculus

- Tableau rule: $\dfrac{\ell_1 : C_1, \quad \dots, \quad \ell_n : C_n}{\ell^1_1 : D^1_1, \quad \dots, \quad \ell^1_{k_1} : D^1_{k_1} \mid \cdots \mid \ell^m_1 : D^m_1, \quad \dots, \quad \ell^m_{k_m} : D^m_{k_m}}$.
- A *clash* rule is a tableau rule where $m = 0$.
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- Given a concept C, tableau $T(C)$ is a (completely) expanded tree of sets of concepts such that
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Common Tableau Rules

Standard rules for $\mathcal{ALC}$

$$(\bot)\ \frac{\ell : C, \quad \ell : \neg C}{\bot} \qquad (\neg\neg)\ \frac{\ell : \neg\neg C}{\ell : C}$$

$$(\neg\sqcup)\ \frac{\ell : \neg(C \sqcup D)}{\ell : \neg C, \quad \ell : \neg D} \qquad (\sqcup)\ \frac{\ell : (C \sqcup D)}{\ell : C \mid \ell : D}$$

$$(\exists)\ \frac{\ell : \exists R.C}{\ell : \exists R.\{\ell'\}, \quad \ell' : C}(\ell' \text{ is new}) \qquad (\neg\exists)\ \frac{\ell : \neg\exists R.C, \quad \ell : \exists R.\{\ell'\}}{\ell' : \neg C}$$

Rules for individuals

$$(\text{sym})\ \frac{\ell : \{\ell'\}}{\ell' : \{\ell\}} \qquad (\neg\text{sym})\ \frac{\ell : \neg\{\ell'\}}{\ell' : \neg\{\ell\}} \qquad (\text{ref})\ \frac{\ell : C}{\ell : \{\ell\}}$$

$$(\text{mon})\ \frac{\ell : \{\ell'\}, \quad \ell' : C}{\ell : C} \qquad (\text{canc})\ \frac{\ell : (\ell' : C)}{\ell' : C}$$

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$\ell : \{\ell'\} \equiv \ell = \ell'$

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Unrestricted Blocking Rule

$$\text{(ub)}\ \frac{\ell : \{\ell\},\quad \ell' : \{\ell'\}}{\ell : \{\ell'\} \mid \ell : \neg\{\ell'\}}$$

Strategy conditions:

1. any rule is applied at most once to the same set of premises.
2. the (∃) rule is not applied to role assertion expressions.
3. if $\ell : \{\ell'\}$ in current branch and $\ell < \ell'$ then no applications of the (∃) rule to expressions $\ell' : \exists R.C$ are performed[1]
4. in every open branch there is some node from which point onwards, all possible applications of the (ub) rule have been performed before any application of the (∃) rule

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[1] $<$ reflects the order in which the individuals are introduced

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Constructive Completeness and sub-Compatibility

Let $\mathcal{B}$ be an open branch in a tableau.

- $\ell \sim_{\mathcal{B}} \ell' \overset{\text{def}}{\Longleftrightarrow} \ell : \{\ell'\} \in \mathcal{B}$,
- $\Delta^{\mathcal{I}(\mathcal{B})} \overset{\text{def}}{=} \{\|\ell\| \mid \ell : \{\ell\} \in \mathcal{B}\}$.

A tableau calculus T_L is *constructively complete* for L iff for any satisfiable concept C and any open branch $\mathcal{B}$ in $T_L(C)$ there is an L-model $\mathcal{I}(\mathcal{B}) = (\Delta^{\mathcal{I}(\mathcal{B})}, \cdot^{\mathcal{I}(\mathcal{B})})$ such that

- $\ell : D \in \mathcal{B}$ implies $\|\ell\| \in D^{\mathcal{I}(\mathcal{B})}$, and
- $\ell : \exists R.\{\ell'\} \in \mathcal{B}$ implies $(\|\ell\|, \|\ell'\|) \in R^{\mathcal{I}(\mathcal{B})}$.

T_L is *compatible with* sub iff for any concept C and $\ell : D$ in $T_L(C)$ either

- $D \in \text{sub}(C)$, or
- $D = \{\ell'\}$, or $D = \neg\{\ell'\}$, or
- $D = \exists R.\{\ell'\}$, or $D = \neg\exists R.\{\ell'\}$, for some role $R \in \text{sub}(C)$.

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Let $\mathcal{B}$ be an open branch in a tableau.

- $\ell \sim_{\mathcal{B}} \ell' \overset{\text{def}}{\Longleftrightarrow} \ell : \{\ell'\} \in \mathcal{B}$,
- $\Delta^{\mathcal{I}(\mathcal{B})} \overset{\text{def}}{=} \{\|\ell\| \mid \ell : \{\ell\} \in \mathcal{B}\}$.

A tableau calculus T_L is *constructively complete* for L iff for any satisfiable concept C and any open branch $\mathcal{B}$ in $T_L(C)$ there is an L-model $\mathcal{I}(\mathcal{B}) = (\Delta^{\mathcal{I}(\mathcal{B})}, .^{\mathcal{I}(\mathcal{B})})$ such that

- $\ell : D \in \mathcal{B}$ implies $\|\ell\| \in D^{\mathcal{I}(\mathcal{B})}$, and
- $\ell : \exists R.\{\ell'\} \in \mathcal{B}$ implies $(\|\ell\|, \|\ell'\|) \in R^{\mathcal{I}(\mathcal{B})}$.

T_L is *compatible with* sub iff for any concept C and $\ell : D$ in $T_L(C)$ either

- $D \in \text{sub}(C)$, or
- $D = \{\ell'\}$, or $D = \neg\{\ell'\}$, or
- $D = \exists R.\{\ell'\}$, or $D = \neg\exists R.\{\ell'\}$, for some role $R \in \text{sub}(C)$.

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The Main Theorem

Theorem

Let L be a (description) logic. $T_L +$ (ub) is sound, complete, and terminating tableau calculus for L, if the following conditions all hold:

1. sub *is a finite closure operator for L-expressions.*
2. *L is a logic which admits finite filtration.*
3. *T_L is a sound and constructively complete tableau calculus for L and is compatible with* sub.

Sound and Constructively Complete Tableau Calculus for $\mathcal{SO}$

$T_{\mathcal{SO}}$ contains the common tableau rules and the following rules for every $s \in$ Trans:

$$(\mathsf{Trans}_s)\ \frac{\ell : \exists s.\{\ell'\},\quad \ell' : \exists s.\{\ell''\}}{\ell : \exists s.\{\ell''\}}$$

- Soundness is trivial.
- Constructive completeness is easy.
- Clearly, $T_{\mathcal{SO}}$ is compatible with the subexpression operator sub.

Theorem

$T_{\mathcal{SO}}$ + (ub) *is sound, complete, and terminating.*

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Sound and Constructively Complete Tableau Calculi for $\mathcal{ALBO}$

$T_{\mathcal{ALBO}}$ contains the common tableau rules and the following rules for complex role operators:

Positive Role Occurrences

$$(\exists\sqcup)\ \frac{\ell : \exists(R \sqcup S).\{\ell'\}}{\ell : \exists R.\{\ell'\} \mid \ell : \exists S.\{\ell'\}}$$

$$(\exists^{-1})\ \frac{\ell : \exists R^{-1}.\{\ell'\}}{\ell' : \exists R.\{\ell\}}$$

$$(\exists\neg)\ \frac{\ell : \exists\neg R.\{\ell'\}}{\ell : \neg\exists R.\{\ell'\}}$$

Negative Role Occurrences

$$(\neg\exists\sqcup)\ \frac{\ell : \neg\exists(R \sqcup S).C}{\ell : \neg\exists R.C,\ \ell : \neg\exists S.C}$$

$$(\neg\exists^{-1})\ \frac{\ell : \neg\exists R^{-1}.C,\ \ell' : \exists R.\{\ell\}}{\ell' : \neg C}$$

$$(\neg\exists\neg)\ \frac{\ell : \neg\exists\neg R.C,\ \ell' : \{\ell'\}}{\ell : \exists R.\{\ell'\} \mid \ell' : \neg C}$$

$T^q_{\mathcal{ALBO}} \stackrel{\text{def}}{=} T_{\mathcal{ALBO}} - (\exists\neg)$

- Both calculi are sound and compatible with the subexpression operator sub.
- $T_{\mathcal{ALBO}}$ is constructively complete w.r.t. $\mathcal{ALBO}$-models.
- $T^q_{\mathcal{ALBO}}$ is constructively complete w.r.t. $\mathcal{ALBO}$-quasi-models.

Theorem

- $T_{\mathcal{ALBO}} +$ (ub) *is sound, complete w.r.t. $\mathcal{ALBO}$-models, and terminating.*
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Outline

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Conclusion

- A general method for turning ground semantic tableau calculi into decision procedures is introduced.
- The method is illustrated on two examples: $\mathcal{SO}$ and $\mathcal{ALBO}$.
- The method is not limited by description logic language.
- It works for other ground tableau and similar decision approaches.
- The framework provides a basis for enhancing prover engineering platforms with a flexible blocking mechanism with which more general tableau decision procedures can be constructed.
- The approach also provides the theoretical background for the way blocking is implemented in the METTEL system.
- The framework is a first step towards the ambitious goal of automated generation of provers for decidable logics.

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