Blocking Mechanisms in Description Logics, A General Approach

Dmitry Tishkovsky

School of Computer Science
The University of Manchester
dmitry.tishkovsky@manchester.ac.uk

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Outline

1. Crash course on DLs
   - ALC
   - Universal modality
   - TBox
   - ABox and individuals
   - RBox
   - Problem statement
   - Tableau procedure
2. Blocking mechanisms
   - Subset blocking
   - Equality blocking
   - Pairwise blocking
   - Successor and anywhere blocking
3. General blocking mechanism
   - Unrestricted blocking rule
   - Simulating existing blocking mechanisms

$\mathcal{ALC}$ Syntax and Semantics

Atomic concepts: $p, p_0, p_1 \ldots$

Atomic roles: $r, r_0, r_1 \ldots$

Concepts: $C, D \stackrel{\text{def}}{=} p \mid \neg C \mid C \sqcup D \mid \exists r.C$

$$C \sqcap D \stackrel{\text{def}}{=} \neg(\neg C \sqcup \neg D), \qquad \forall r.C \stackrel{\text{def}}{=} \neg \exists r.\neg C.$$

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Interpretation (model): $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ satisfying

$$p^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \qquad r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \qquad \ell^{\mathcal{I}} \in \Delta^{\mathcal{I}}$$

$$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} \qquad (C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$$

$$(\exists r.C)^{\mathcal{I}} = \{x \mid \exists y \in C^{\mathcal{I}}\ (x, y) \in r^{\mathcal{I}}\}$$

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Universal Modality

- $\forall C$, $\exists C$.
- $(\forall C)^{\mathcal{I}} \stackrel{\text{def}}{=} \{x \mid \forall y \in S\ y \in C^{\mathcal{I}}\}$.
- $(\exists C)^{\mathcal{I}} \stackrel{\text{def}}{=} \{x \mid \exists y \in S\ y \in C^{\mathcal{I}}\}$.

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General TBox

- *Terminological axiom:* $C = D$.
- Interpretation: $C^{\mathcal{I}} = D^{\mathcal{I}}$.
- A *general TBox* is a set of terminological axioms.
- *Subsumption axiom* $C \sqsubseteq D$.
- Interpretation: $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$.
- Every general TBox is equivalent to a set of subsumption axioms.
- If language contains a universal modality than TBox is representable as a set of concepts: $\forall(\neg C \sqcup D)$.

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ABox and Individuals

- ABox A is a set of *concept assertions* $C(a)$ and *role assertions* $R(a, b)$ where a, b stand for an elements of a model.
- *Set of individual names:* $\ell, \ell_0, \ell_1, \ldots$
- *Singleton concepts:* $\{\ell\}$.
- Interpretation: $\ell^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ and $\{\ell\}^{\mathcal{I}} \stackrel{\text{def}}{=} \{\ell^{\mathcal{I}}\}$.
- *Concept assertion:* $\ell : C$.
- Interpretation:

$$(\ell : C)^{\mathcal{I}} \stackrel{\text{def}}{=} \begin{cases} \Delta^{\mathcal{I}}, & \text{if } \ell^{\mathcal{I}} \in C^{\mathcal{I}}, \\ \varnothing, & \text{otherwise,} \end{cases}$$

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RBox

- RBox is a set of *role inclusion axioms* $R \sqsubseteq S$.
- possibly, other assumptions on roles are included, e.g. transitivity of roles.

Problem Statement

- *Knowledge base* KB is a tuple (ABox, TBox, RBox).
- The problem (KB $\models C$?):
  Given a knowledge base KB and a concept C, find a model $\mathcal{I}$ which validate all the axioms of the knowledge base and $C^{\mathcal{I}} \neq \varnothing$.
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Tableau Procedure
Common Tableau Rules

Standard rules for $\mathcal{ALC}$

$$(\bot)\ \frac{\ell : C,\quad \ell : \neg C}{\bot} \qquad (\neg\neg)\ \frac{\ell : \neg\neg C}{\ell : C}$$

$$(\neg\sqcup)\ \frac{\ell : \neg(C \sqcup D)}{\ell : \neg C,\quad \ell : \neg D} \qquad (\sqcup)\ \frac{\ell : (C \sqcup D)}{\ell : C \mid \ell : D}$$

$$(\exists)\ \frac{\ell : \exists R.C}{\ell : \exists R.\{\ell'\},\quad \ell' : C}(\ell' \text{ is new}) \qquad (\neg\exists)\ \frac{\ell : \neg\exists R.C,\quad \ell : \exists R.\{\ell'\}}{\ell' : \neg C}$$

Rules for individuals

$$(\text{sym})\ \frac{\ell : \{\ell'\}}{\ell' : \{\ell\}} \qquad (\neg\text{sym})\ \frac{\ell : \neg\{\ell'\}}{\ell' : \neg\{\ell\}} \qquad (\text{ref})\ \frac{\ell : C}{\ell : \{\ell\}}$$

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Blocking

- Blocking is a detection of repetitions in partially constructed models.
- If some conditions are true for given two individuals (labels) ℓ and ℓ' then ℓ is blocked by ℓ' (for application of individual generating rules).
- To avoid cyclic blocking we assume that all individuals in branch are linearly ordered by an ordering $<$ and given two nominals we always block the largest one w.r.t. the ordering.
- If blocks on individuals are never undone then blocking is *static*. Otherwise it is called *dynamic*.
- Blocking mechanisms usually require access to a set of concepts $\tau(\ell)$ associated with given individual ℓ and sometimes a set of role links $\tau(\ell, \ell')$ associated with two individuals within the same branch $\mathcal{B}$:

$$\tau(\ell) \stackrel{\text{def}}{=} \{C \mid \ell : C \in \mathcal{B}\}$$

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Subset Blocking

- If $\tau(\ell) \subseteq \tau(\ell')$ (and $\ell' < \ell$) then ℓ is blocked by ℓ'.
- Example: TBox $= \{\top \sqsubseteq \exists r.p\}$, a concept $p \sqcap q$.

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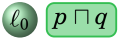

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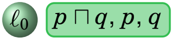

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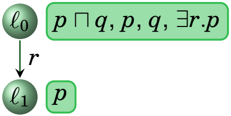

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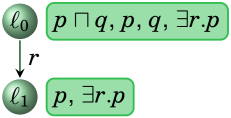

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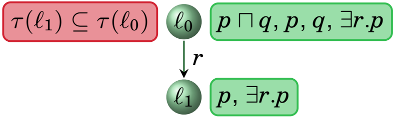

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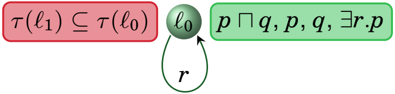

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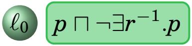

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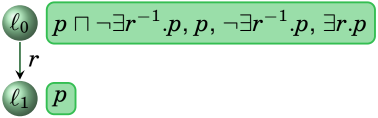

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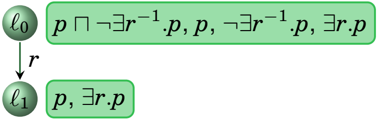

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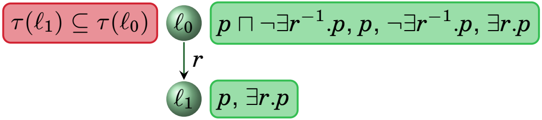

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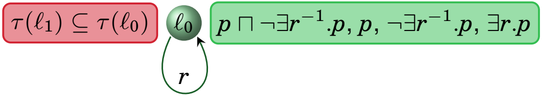

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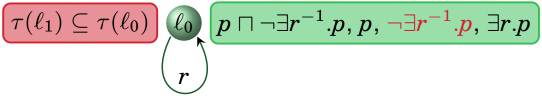

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$$\ell_0 \quad p \sqcap \neg\exists r^{-1}.p,\ p,\ \neg\exists r^{-1}.p,\ \exists r.p$$

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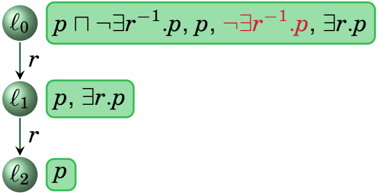

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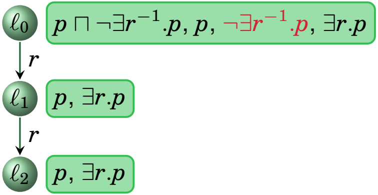

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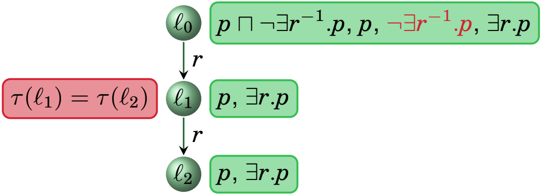

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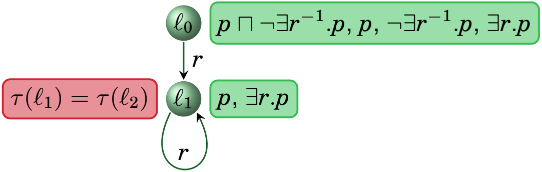

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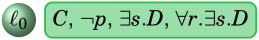

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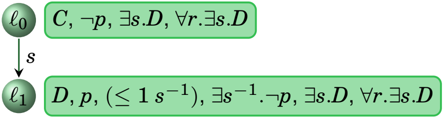

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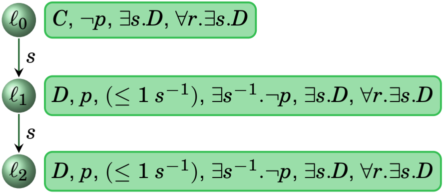

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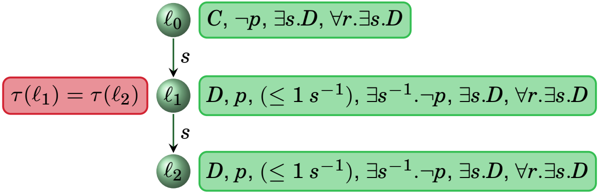

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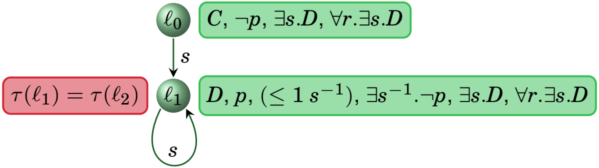

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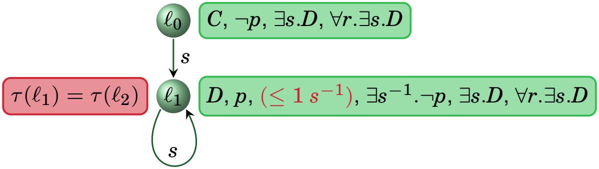

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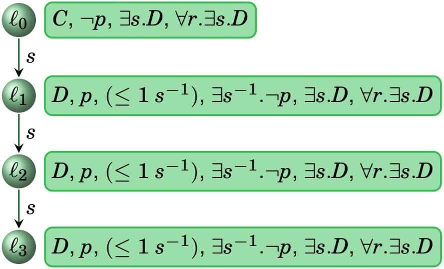

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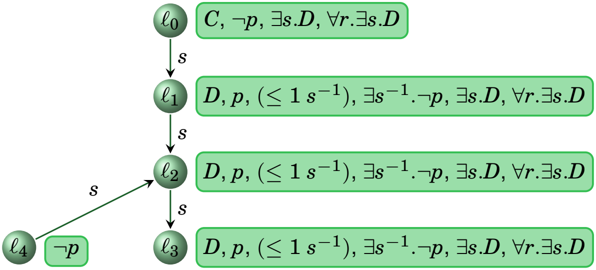

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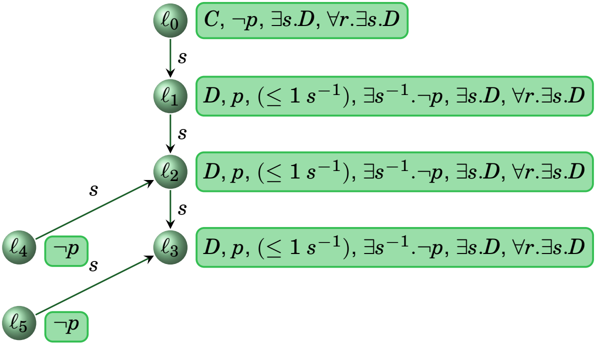

Successor and Anywhere Blocking

Successor blocking:

- Block ℓ by ℓ' only if ℓ is a successor of ℓ' along some path of role links in the branch.
- It is sufficient for logics which have the tree-model property.

Anywhere blocking:

- Blocks are allowed for any pair of individuals in given branch.

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Outline

1 Crash course on DLs
- ALC
- Universal modality
- TBox
- ABox and individuals
- RBox
- Problem statement
- Tableau procedure

2 Blocking mechanisms
- Subset blocking
- Equality blocking
- Pairwise blocking
- Successor and anywhere blocking

3 General blocking mechanism
- Unrestricted blocking rule
- Simulating existing blocking mechanisms

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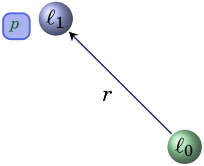
Blocking Problem for $\mathcal{ALBO}$

ℓ_0

$\exists r.p$
$\exists s.p$
$\exists t.\neg\exists t.\exists s^{-1}.\top$
$\neg\exists t.\exists\neg t.\neg\exists s^{-1}.\top$

Blocking Problem for $\mathcal{ALBO}$

$$\frac{\ell : \exists R.C}{\ell : \exists R.\{\ell'\}, \quad \ell' : C}$$



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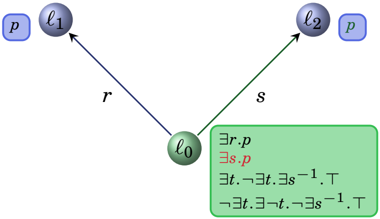

$\ell_1 = \ell_2$?

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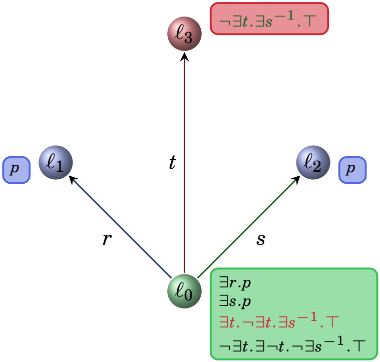

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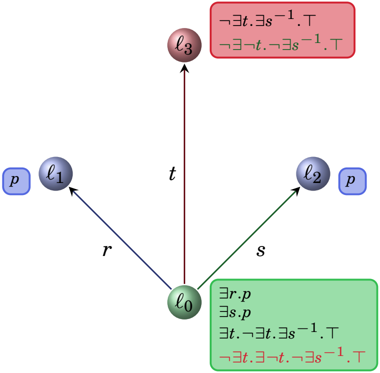

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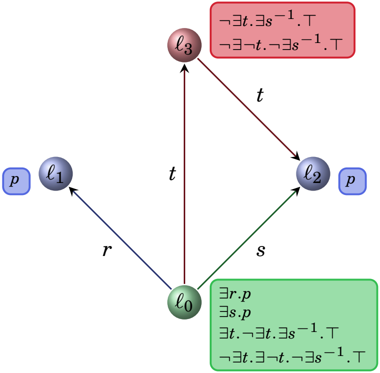

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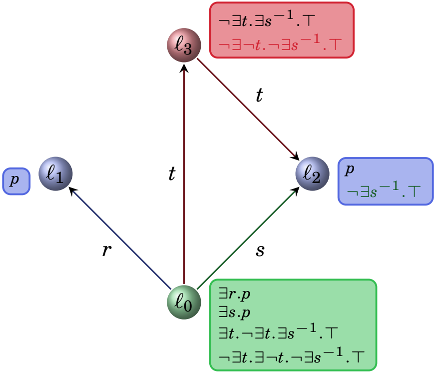

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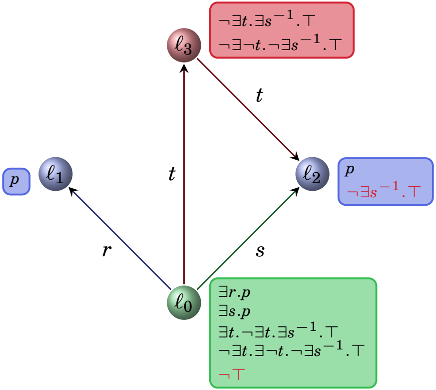

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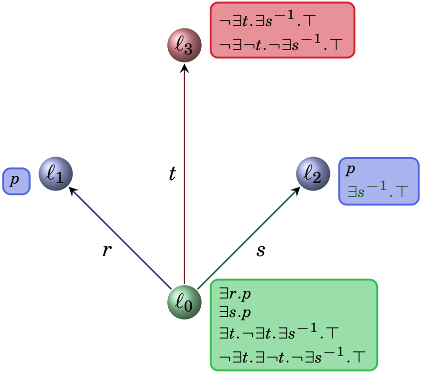

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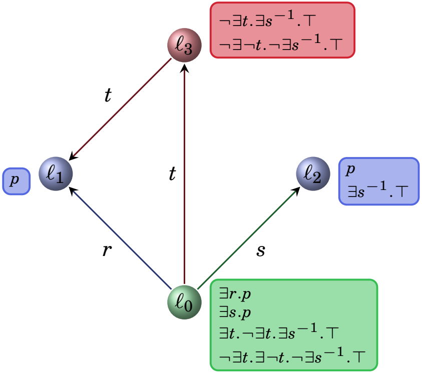

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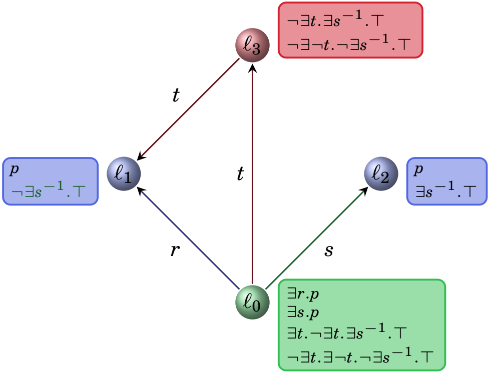

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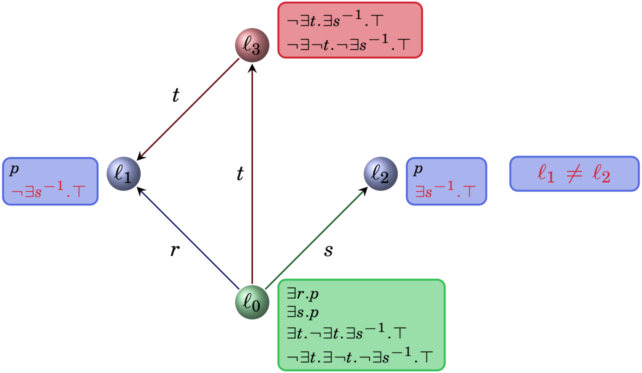

Unrestricted Blocking Rule

$$\text{(ub)}\ \frac{\ell : \{\ell\},\quad \ell' : \{\ell'\}}{\ell : \{\ell'\} \mid \ell : \neg\{\ell'\}}$$

Strategy conditions:

1. any rule is applied at most once to the same set of premises.
2. the (∃) rule is not applied to role assertion expressions.
3. if $\ell : \{\ell'\}$ in current branch and $\ell < \ell'$ then no applications of the (∃) rule to expressions $\ell' : \exists R.C$ are performed[1]
4. in every open branch there is some node from which point onwards, all possible applications of the (ub) rule have been performed before any application of the (∃) rule

[1] $<$ reflects the order in which the individuals are introduced

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Simulating Existing Blocking Mechanisms

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Simulating Existing Blocking Mechanisms

- Add conditions for blocking as constraints on application of the unrestricted blocking rule.
- Ensure that tableau uses a fair strategy.
- Ensure the condition 4.
- Tableau algorithm is guaranteed to terminate for logics with the effective finite model property.

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