An analytical study of the effect of hydrodynamic mixing on the photosynthetic microorganism growth

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1 Introduction

The photosynthetic microorganism growth description is usually based on the so-called microbial kinetics, i.e. on the lumped parameter models (LPM) describing the photosynthetic response in small cultivation systems with a homogeneous light distribution [3, 6]. However, there is an important phenomenon, the so-called flashing light enhancement, which demands some other model than it residing in the artificial connection between the steady state kinetic model and the empiric one describing the photosynthetic productivity under fluctuating light condition. Nevertheless, even having an adequate dynamical LPM of microorganism growth, see e.g. phenomenological model of so-called photosynthetic factory [4, 5], another serious difficulty resides in the description of the microalgal growth in a photobioreactor (PBR), i.e. in a distributed parameter system.

In order to develop the distributed parameter model (DPM) of a microorganism growth, two main approaches for transport and bioreaction processes modelling are usually chosen: (i) Eulerian infinitesimal, and (ii) Eulerian multicompartmental. While the Eulerian infinitesimal approach, leading to the partial differential equations (PDE), is an usual way to describe transport and reaction systems, the multicompartmental modelling framework, resulting in a system of ordinary differential equations (ODE), is mostly used in the process engineering area. This second approach, based on balance equation among compartments with finite control volume, has been recently treated by Bezzo *et al.* [2]. The authors presented there a rigorous mathematical framework for constructing *hybrid multicompartment/CFD models. Hybrid* there means that the fluid flow description is resolved by a CFD code, and does not make a part of the ODE system of governing equations.

In the sequel, we adopt the first approach aiming to clarify in an analytical manner the role of hydrodynamic mixing, or more precisely, the mechanism of the photosynthetic microorganism growth enhancement due to the microbial cell transport by radial dispersion. Nevertheless, in the future work, our results should serve to develop a numerical scheme for setting up the optimal compartment size in the multicompartment/CFD models.

2 Model development

Accordingly to [7], the transport equation for microbial cells (concentration or cell density c) as the function of spatial coordinates and time gets the next form:

$$\frac{\partial c}{\partial t} + \nabla \cdot (\vec{v}c) - \nabla \cdot (D_e \nabla c) = R \ , \qquad (1)$$

where R is the source term (representing microbial growth, unit: cell m^{-3}s^{-1}), $\vec{v}$ represents the velocity field, and D_e is the dispersion coefficient, which corresponds to diffusion coefficient in microstructure description and becomes mere empirical parameter suitably describing mixing in the system. D_e is influenced by the molecular diffusion and velocity profile. When mixing is mainly caused by the turbulent micro-eddies, the phenomenon is called the turbulent diffusion and a *turbulent diffusion coefficient* is introduced e.g. in [1]. The reaction obviously depends on some variables, usually called as substrates. For our special case of photosynthetic growth in a PBR, the role of only one limiting substrate (the nutrients are supposed to be present in a sufficient amount, i.e. they do not limit the growth) fulfills the irradiance, in other words, an external forcing input u. Moreover we suppose the rectangular PBR geometry illuminated from one side, i.e. the irradiance level is decreasing from the PBR wall to PBR core. Thus, the PBR volume (our computational domain) can be divided into layers with the same irradiance level, transforming the 3D problem into the one-dimensional. Consequently, the description of cell motion in direction of light gradient, i.e. perpendicular to PBR wall and at the same time perpendicular to the direction of convective flow, is of most interest. This motion is caused by the just mentioned turbulent diffusion. Furthermore, we can introduce the dimensionless spatial coordinate x by $r := xL$, where L is the the PBR length in direction of light gradient, and the dimensionless dispersion coefficient $p(x)$ by $D_e := p(x)\ D_0$, where D_0 is a constant with some characteristic value, unit: $\mathrm{m^2s^{-1}}$. Furthermore we introduce the dimensionless concentrations as $y := \frac{c}{c_m}$, $y_{ss} := \frac{c_{ss}}{c_m}$, where c_m is a characteristic (e.g. maximal) concentration of c.

Based on photosynthetic factory model [4, 5] we have for the reaction term R the relation

$$R = -k\ (c - c_{ss})\ , \tag{2}$$

where k is the rate (unit: $\mathrm{s^{-1}}$) associated with the dynamic process by which is the concentration approaching to some value c_{ss} depending only on the external input u.

As we are interested on the steady state solution of (1), i.e. $\frac{\partial c}{\partial t} = 0$, we obtain

$$-\left[p(x)y'\right]' + q(x)\ y = q(x)\ y_{ss}, \quad y'(0) = 0,\ y'(1) = 0\ , \tag{3}$$

where $q(x) := \frac{k(u(x))\ L^2}{D_0}$.

If we define k_0 as follows: $k := k_A(u(x))\ k_0$, then the characteristic number, so-called *Damköhler number* of second type, could be defined as $Da_{II} := \frac{k_0 L^2}{D_0}$, and the the dependence of the solution of (3) on Da_{II} could be studied.

3 Analytical solution

In fact, we do not need the solution of equation (3) in form $y = y(x)$, but we want to find the mean value of y in the interval $x \in [0.1]$, i.e. to compute the expression $\int_0^1 y(x)\ \mathrm{d}x$. Based on [8], the boundary value problem is transformed into the related initial value problem. It consists in finding solutions of two homogeneous equations, two differential equations with the right-hand side and computing a solution of a system of two algebraic equations. The result is that we obtain a function value and its derivative in an arbitrary point. The original differential equation with boundary conditions is thus transformed into a differential equation with an initial condition. As we need only a solution in several points, we can apply the above procedure repeatedly. Finally, the value $\int_0^1 y(x)\ \mathrm{d}x$ is obtained by a suitable numerical method.

4 Conclusion

An analytical study of the effect of hydrodynamic mixing on the photosynthetic microorganism growth is presented. The spatio-temporal dependence of microorganism cell concentration in our system of interest, i.e. in the photobioreactor (PBR), is reduced into a one-dimensional problem described by the second-order non-homogeneous ordinary differential equation with the non-linear continuous function on the right hand side. The impermeability of the PBR's walls imposes the Neumann boundary condition. The related initial value method is applied, and for a special case of forcing input and for the special right hand side (Haldane type kinetics), the resulting dependence of the PBR productivity (the average value of steady-state concentration) on hydrodynamic mixing is determined.

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