Triangular updates for solving sequences of linear algebraic systems

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We consider the solution of sequences of linear systems

$$A^{(i)}x = b^{(i)},\ i = 1, \ldots,$$

where $A^{(i)} \in \mathbb{R}^{n\times n}$ are general nonsingular sparse matrices and $b^{(i)} \in \mathbb{R}^n$ are corresponding right-hand sides. Such sequences arise in many applications like computational fluid dynamics, structural mechanics, numerical optimization as well as in solving non-PDE problems. For example, a system of nonlinear equations $F(x) = 0$ for $F : \mathbb{R}^n \to \mathbb{R}^n$ solved by a Newton or Broyden-type method leads to a sequence of problems

$$J(x_i)(x_{i+1} - x_i) = -F(x_i),\ i = 1, \ldots, \tag{1}$$

where $J(x_i)$ is the Jacobian evaluated in the current iteration x_i or its approximation.

The solution of such sequences of linear systems is often one of the main bottlenecks in applications. Preconditioned iterative Krylov subspace solvers are often methods of choice when the systems are large. Computing preconditioners $M^{(1)}, M^{(2)}, \ldots$ for individual systems separately may be rather expensive in many practical situations, in particular in matrix-free or parallel computational environment. There is a strong need for reduction of costs by sharing some of the computational effort among the subsequent linear systems.

Our contribution is targeted to improvements of algebraic preconditioners for solving subsequent members of a sequence. Here we will discuss *approximate* triangular updates [1], [2], [3] of a preconditioner $M^{(1)}$ factorized by $LDU \approx A$ decomposition. We will concentrate on solving the problem in matrix-free environment [4] and we will discuss two different approaches. Efficiency of the new approaches will be demonstrated by numerical experiments.

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References

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