Noise propagation in Golub-Kahan bidiagonalization and its relation to projection methods for solving discrete inverse problems

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Abstract.

A broad class of application requires solving discrete inverse problems of the form

$$b = Ax + e,$$

where e is an unknown perturbation of the exact data, $\|e\| \ll \|Ax\|$. In many cases, A represents a discretized *smoothing* operator with singular values decaying gradually to zero (e.g. a discretized blurring operator in image deblurring problem). In such cases, the Picard condition is often violated and the naive solution $A^\dagger b$, where $A^\dagger$ denotes the Moore-Penrose pseudoinverse, is a meaningless noise-dominated vector. Therefore, to find a reasonable approximation of the exact solution x, regularization techniques have to be employed. A successful regularization method has to suppress the devastating effect of noise while preserving sufficient information from the data.

The Golub-Kahan iterative bidiagonalization belongs to popular techniques with regularization properties. It projects the original problem to a lower dimensional (Krylov) subspace. In [1] it was shown that, due to the smoothing properties of multiplication by A and A^T and the orthogonality that is locally maintained among the bidiagonalization vectors, the bidiagonalization also has ability to reveal the high-frequency part of the noise vector e in its left bidiagonalization vectors.

In this contribution, we restrict ourselves to problems corrupted by white noise, which is a natural assumption in many applications. We will show how can the noise vector e be estimated based on the results of [1] and we will propose a way to regularize the inverse problem by subtracting this estimate from the noisy data b, as described in [2]. This procedure of noise reduction is closely related to standard projection methods involving Golub-Kahan bidiagonalization, which gives an alternative insight into the problem of iterative regularization. As a side product, it provides a useful stopping criterion that can be computed at a negligible cost.

Referencias

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