Aiding an Introduction to Formal Reasoning Within a First-Year Logic Course for CS Majors Using a Mobile Self-Study App

David M. Cerna, Martina Seidl, Wolfgang Schreiner, Wolfgang Windsteiger, and Armin Biere

JOHANNES KEPLER UNIVERSITY LINZ

June 17^{th} 2020

FMV

Introduction

- We form the core team of the LOGTECHEDU project.

  http://fmv.jku.at/logtechedu/
- The goals of this project are:
  - Development and use of educational logic-based software tools.
  - Expanding logic education within undergraduate CS curricula.
- Our introductory logic course exemplifies these points:
  - The course is **heavily supported** by software.
- In this talk we:
  - Outline our introductory logic **course**,
  - introduce **AX**olotl, one of the educational tools we developed,
  - and discuss an **experiment** testing the benefits of **AX**olotl as a study aid for **inferential reasoning**.

Logic and Computer Science

- Quoting [Makowsky and Zamansky, 2017], the presence of logic courses in undergraduate university curricula is in decline.
- This trend has nothing to do with the relevance of logic to modern computer science.
- Verification technology relies on automated reasoners that evaluate logical formulas [Calcagno et al. , 2015; Cook, 2018].
- Logical formalisms form the basis of the symbolic branch of artificial intelligence [Russell and Norvig, 2010].
- If anything, **logic** is as important to computer science today as it was at the foundation of the field.
- Our view: **logic** is a foundational subject of **CS**, not an advanced elective.

Logic: A First Semester Course

- We made logic one of the first things new CS students see.
- As one may expect students will not be provided a detailed exposition of the **deeper aspects of mathematical logic**.
- What they will see:
  - Encoding of problems as SAT and SMT formulas.
  - An introduction to formal language.
  - Syntax and semantics separation.
  - Construction of formal proofs.
- Throughout the semester, **educational software** is used to aid understanding and provide practical use cases.
- Some of the course material may be found here:

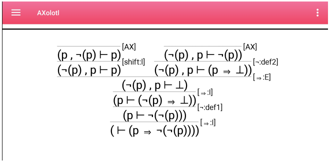
http://fmv.jku.at/logic/index.html

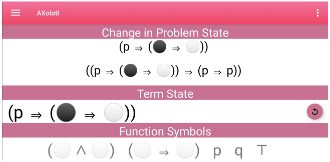
Logic Course: Structure

- The course concise of three modules, namely
  - Propositional Logic or SAT (4 weeks)
  - First-order logic or FOL (6 weeks)
  - SMT (2 weeks)
- Every week there is a **mini-test** examining students.
- **Weekly challenges** provide bonus points for the next test.
- Each module has **lab assignments**, FOL has two.
- Lab assignments may replace a mini-test from its module.
- Both weekly challenges and lab assignments are **optional** and require the use of our **educational software**.
- The **first lab assignment** introduced **AX**olotl as a study tool for **inferential reasoning** and **proof construction**.
  - **AX**olotl https://play.google.com/store/apps/details?id=org.axolotlLogicSoftware.axolotl

Software: **AX**olotl

- An Android app available on google play.
- Aids students through formal rule application and proof construction.
- Educators can make new problems using a simple input language.
- Designed for a fragment of quantifier-free first order logic.
- A work in progress. Improvement and expansion are planned.





Inference rules in **AX**olotl

- An inference rule has the following form in AXolotl:

$$\Delta, E_1 \Rightarrow \Delta, E_2, \cdots, E_n$$

  - Δ is a list of expressions which remain unchanged by the inference rule.
  - E_1 is a single expression, the target of the inference rule.
  - $E_2, \cdots, E_n$ denote the results of the inference rule.

$$\Delta, (x, y \vdash x, z) \Rightarrow \Delta \qquad \text{Axiom}$$

$$\Delta, x \circ (y \circ z) \Rightarrow \Delta, (x \circ y) \circ z \qquad \text{Associativity}$$

$$\Delta, ((x \rightarrow y), z \vdash w) \Rightarrow \Delta, (y, z \vdash w), (z \vdash x, w) \qquad \text{Implication}$$

$$\Delta, (z \vdash x) \Rightarrow \Delta, (\neg(x), z \vdash \bot) \qquad \text{Contradiction}$$

- Variables are placeholders for expressions.

AXolotl File: Propositional Sequent Calculus

```
Function: ⇒ 2 infix
Function: ∧ 2 infix
Function: ∨ 2 infix
```

Logical Operators

```
Function: p 0
Function: q 0
Function: r 0
```

Propositional Constants

```
Variable: x
Variable: y
Variable: z
Variable: w
```

Propositional Variables

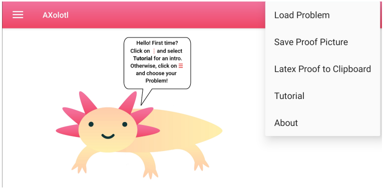
```
Problem: 1 ⊢(∧(p,∨(q,r)),∨(∧(p,q),∧(p,r)))
```

Problem Statement

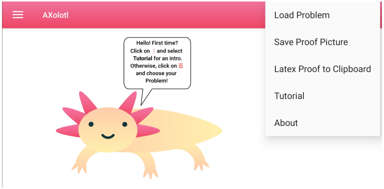
```
Rule: 1 ⊢(w,cons(x,cons(y,z))) ⊢(w,cons(∨(x,y),z)) [∨:r]
Rule: 2 ⊢(cons(x,z),w) ⊢(cons(y,z),w) ⊢(cons(∨(x,y),z),w) [∨:l]
Rule: 2 ⊢(w,cons(x,z)) ⊢(w,cons(y,z)) ⊢(w,cons(∧(x,y),z)) [∧:r]
Rule: 1 ⊢(cons(x,cons(y,z)),w) ⊢(cons(∧(x,y),z),w) [∧:l]
Rule: 1 ⊢(w,cons(cons(y,z),x)) ⊢(w,cons(x,cons(y,z))) [shift:r]
Rule: 1 ⊢(cons(cons(y,z),x),w)  ⊢(cons(x,cons(y,z)),w) [shift:l]
Rule: 0 ⊢(cons(x,y),cons(x,z)) [AX]
```

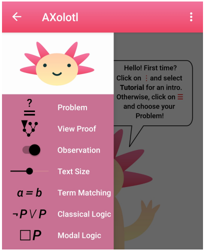
Rules

AXolotl in Action

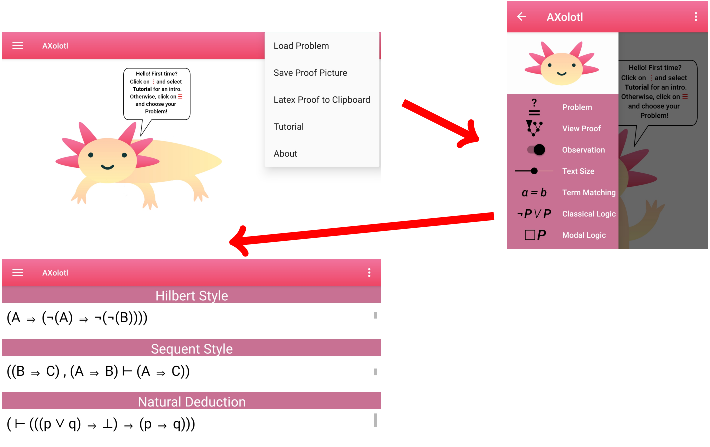


AXolotl in Action

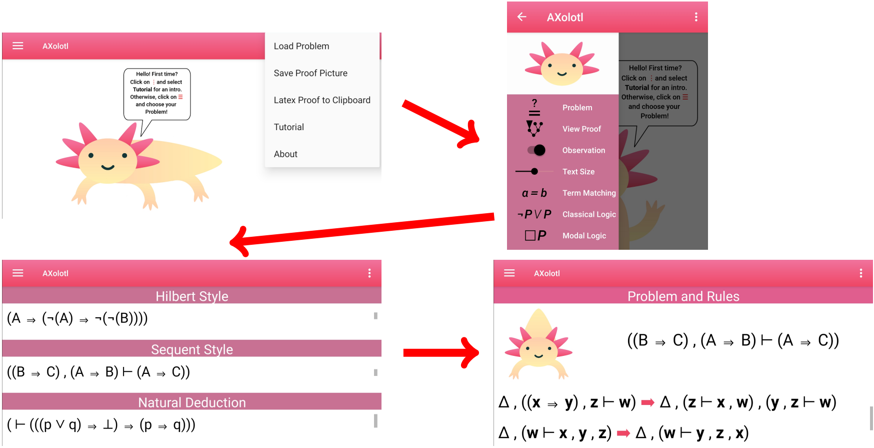




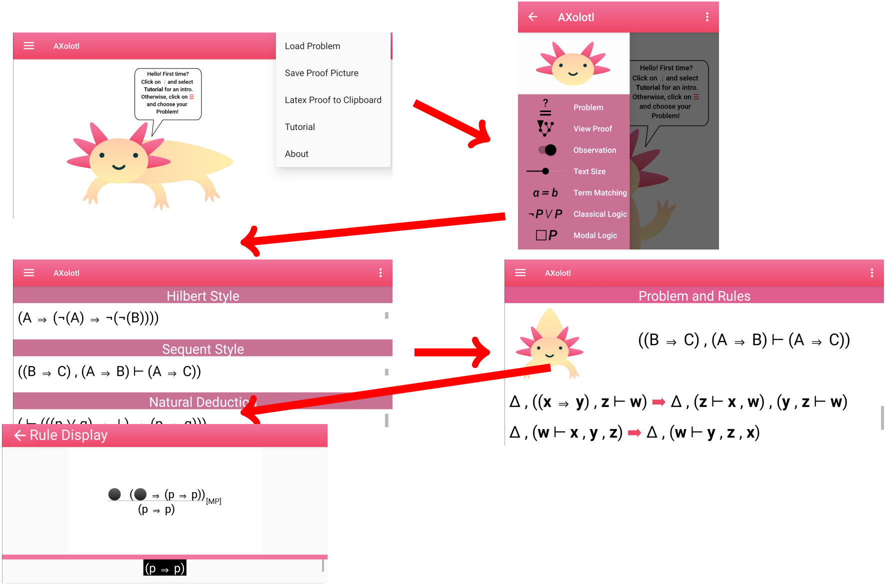
AXolotl in Action



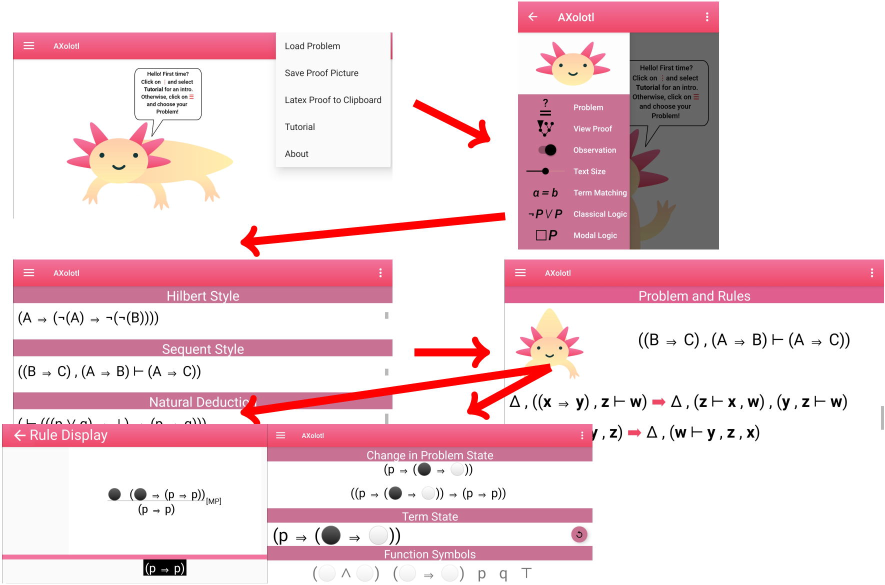
AXolotl in Action



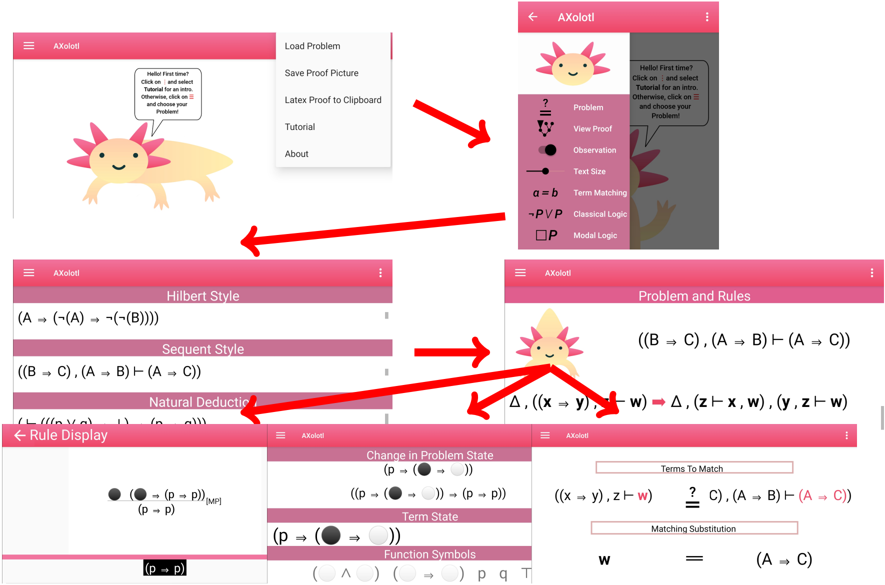
AXolotl in Action



AXolotl in Action



AXolotl in Action



AXolotl in Action

Rules

$$\dfrac{(w \vdash y) \quad (w \vdash (y \Rightarrow x))}{(w \vdash x)}\,[\Rightarrow\!:\!E] \qquad \dfrac{(\neg(x)\,,\, z \vdash \bot)}{(z \vdash x)}\,[\text{Contra}] \qquad \dfrac{}{(x\,,\, y \vdash x\,,\, z)}\,[\text{AX}]$$

$$\dfrac{(z \vdash (x \Rightarrow \bot))}{(z \vdash \neg(x))}\,[\neg\!:\!\text{def1}] \qquad \dfrac{(y\,,\, z\,,\, x \vdash w)}{(x\,,\, y\,,\, z \vdash w)}\,[\text{shift:l}] \qquad \dfrac{(z \vdash \bot)}{(z \vdash x)}\,[\bot\!:\!E]$$

$$\dfrac{(z \vdash \neg(x))}{(z \vdash (x \Rightarrow \bot))}\,[\neg\!:\!\text{def2}] \qquad \dfrac{(x\,,\, z \vdash y)}{(z \vdash (x \Rightarrow y))}\,[\Rightarrow\!:\!I]$$

Proof

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{(p\,,\,(\neg(q) \Rightarrow \neg(p))\,,\,\neg(q) \vdash p)}\,[\text{AX}]}{(\neg(q)\,,\,p\,,\,(\neg(q) \Rightarrow \neg(p)) \vdash p)}\,[\text{shift:l}] \quad \dfrac{\dfrac{\dfrac{}{(\neg(q)\,,\,p\,,\,(\neg(q) \Rightarrow \neg(p)) \vdash \neg(q))}\,[\text{AX}] \quad \dfrac{\dfrac{\dfrac{}{((\neg(q) \Rightarrow \neg(p))\,,\,\neg(q)\,,\,p \vdash (\neg(q) \Rightarrow \neg(p)))}\,[\text{AX}]}{(p\,,\,(\neg(q) \Rightarrow \neg(p))\,,\,\neg(q) \vdash (\neg(q) \Rightarrow \neg(p)))}\,[\text{shift:l}]}{(\neg(q)\,,\,p\,,\,(\neg(q) \Rightarrow \neg(p)) \vdash (\neg(q) \Rightarrow \neg(p)))}\,[\text{shift:l}]}{(\neg(q)\,,\,p\,,\,(\neg(q) \Rightarrow \neg(p)) \vdash \neg(p))}\,[\Rightarrow\!:\!E]}{(\neg(q)\,,\,p\,,\,(\neg(q) \Rightarrow \neg(p)) \vdash (p \Rightarrow \bot))}\,[\neg\!:\!\text{def2}]}{(\neg(q)\,,\,p\,,\,(\neg(q) \Rightarrow \neg(p)) \vdash \bot)}\,[\Rightarrow\!:\!E]}{(p\,,\,(\neg(q) \Rightarrow \neg(p)) \vdash q)}\,[\text{Contra}]}{((\neg(q) \Rightarrow \neg(p)) \vdash (p \Rightarrow q))}\,[\Rightarrow\!:\!I]}{(\vdash ((\neg(q) \Rightarrow \neg(p)) \Rightarrow (p \Rightarrow q)))}\,[\Rightarrow\!:\!I]$$

Experiment: Outline

- The lab assignment introduces inferential reasoning through **Natural Deduction**.
- Prior to the lab, only Decompositional reasoning in clausal logic was introduced.
- Labs are **optional** allowing us to separate the class into a test group and control group.
  - Many students do well in the first module, thus most students are not incentivised to participate.
  - The test group concise of well and poorly performing students.
- **Hypothesis:** Students who performed poorly on the mini-test concerning Decompositional reasoning and participated in the lab will perform better on the mini-test concerning inferential reasoning than students who performed poorly and did not participate in the lab.
- **Poorly Performing**: between 40% to 60% on the mini-test.

Experiment: Results

- The control group contained **213** students, the test group **23**.
- **62** students in the control group and **11** in the test group were categorized as poorly performing.
- Our experiment verified the hypothesis; **low significance**.
- Test group 5 points out of **100** better than the control.
- However, 5 of the **11** poorly performing students in the test group received perfect scores on the next mini-test.
- Only 17 of the **62** poorly performing students in the control group received perfect scores.
- Note, that is 45% for the **test** group and 27% for the **control**.
- The experiment was not designed for this performance metric.

Conclusion & Remarks

- While our experiment did not verify our hypothesis to a desired level of significance, it motivates the need for further and deeper investigation.
- The fact that students seemed to benefit from the lab and the use of **AX**olotl points to some positive educational impact.
- **AX**olotl was one of a few approaches used to introduce inferential reasoning.
- This, as well as the low participation may have influenced the statistical significance.
- Feel free to contact me in the near future at:

cernadavid1@gmail.com

Appendix: Lab Assignment

1) **(.5 points)** Assign the appropriate label to each inference:

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{
          \dfrac{
            \dfrac{\Delta, p \vdash p}{\Delta, p \vdash q \to p}\,(h)
          }{\Delta, p \vdash (p \to q) \lor (q \to p)}\,(g)
          \qquad
          \Delta, p \vdash \neg(p \to q) \lor (q \to p)
        }{\Delta, p \vdash \bot}\,(f)
      }{\Delta, p \vdash q}\,(e)
    }{\Delta \vdash p \to q}\,(d)
  }{\neg(p \to q) \lor (q \to p) \vdash (p \to q) \lor (q \to p)}\,(c)
  \qquad
  \Delta \vdash \neg(p \to q) \lor (q \to p)
}{
  \dfrac{\neg(p \to q) \lor (q \to p) \vdash \bot}{\vdash (p \to q) \lor (q \to p)}\,(a)
}\,(b)
$$

(a) ______________________ (e) ______________________

(b) ______________________ (f) ______________________

(c) ______________________ (d) ______________________

(d) ______________________ (h) ______________________

Appendix: Lab Assignment

2) **(.5 points)** Prove $((P \to Q) \land (Q \to R)) \to (P \to R)$ using Natural Deduction, by hand.

3) **(.5 points)** Download the file *EMProblem.txt* which contains the problem $\vdash \neg A \lor A$ together with the appropriate rules and load the file into AXolotl. Using AXolotl, recreate Proof 7(in ND.pdf)

4) **(.5 points)** Download the file *ContrapositiveProblem.txt* which contains the problem $\vdash (\neg q \to \neg p) \to (p \to q)$ together with the appropriate rules. Load the file into AXolotl and solve it. Save an image of the completed proof.

5) **(1 points)**

   a) Replace the line in *ContrapositiveProblem.txt*

   `Problem: 1 ⊢(ϵ,→(→(¬(q),¬(p)),→(p,q)))`

   by the line

   `Problem: 1 ⊢(∨(¬(q),q),→(→(¬(q),¬(p)),→(p,q)))`

   Remove the line

   `Rule: 1 ⊢(cons(¬(x),z),⊥) ⊢(z,x) [⊥]`

   and add the following lines

   ```
   Rule: 1 ⊢(z,x) ⊢(z,∨(x,y)) [∨:I1]
   Rule: 1 ⊢(z,y) ⊢(z,∨(x,y)) [∨:I2]
   Rule: 3 ⊢(cons(x,z),w) ⊢(cons(y,z),w) ⊢(z,∨(x,y)) ⊢(z,w) [∨:E]
   ```

Appendix: Lab Assignment

b) The statement $(\neg q \rightarrow \neg p) \rightarrow (p \rightarrow q)$ it is referred to as the *contraposivity axiom* and can be proven equivalent to the *principle of excluded middle* $\neg p \vee p$. Proving $(\neg q \rightarrow \neg p) \rightarrow (p \rightarrow q) \vdash (\neg q \vee q)$ using the sequent calculus is quite simiple, is it provable using the rule set of *ContrapositiveProblem2.txt* or is some assumption missing? If an assumption is missing what is it? (see the file *ContrapositiveProblem3.txt*)

c) What about adding

```
Rule: 1 ⊢(cons(¬(x),z),⊥) ⊢(z,x) [⊥]
```

to *ContrapositiveProblem3.txt*, does it change the situation? Can be found in *ContrapositiveProblem4.txt*.

d) What is the relationship between contradiction and the principle of excluded middle? Note that contradiction may be written as $(\neg(p) \rightarrow \bot) \rightarrow p$. Prove the following statement

$$(\neg p \vee p) \vdash (\neg(p) \rightarrow \bot) \rightarrow p$$

using the rules in the file *ContrapositiveProblem5.txt*. Save an image of the completed proof.

6) **(2 points)** Prove the following statements concerning equivalence of various logical operators:

a) $\neg\neg p \leftrightarrow p$ (*dn.txt*)

b) $(\neg p \rightarrow q) \rightarrow (p \vee q)$ (*orimp.txt*)

c) $(p \wedge q) \rightarrow \neg(p \rightarrow \neg q)$ (*andimp.txt*)

d) $(p \wedge (q \vee r)) \rightarrow ((p \wedge q) \vee (p \wedge r))$ (*distribution.txt*)

e) $\neg(p \wedge q) \rightarrow (\neg p \vee \neg q)$ (*demorgen.txt*)

Save an image of each completed proof.