The patch C to function choose() from the email
`https://stat.ethz.ch/pipermail/r-devel/2009-December/056177.html`
uses the following two transformations.

Transformation 1

If $n < 0$, patch C uses the transformation

$$\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1} = \frac{n}{k}\binom{k-1-n}{k-1}(-1)^{k-1} \ . \tag{1}$$

The original implementation uses

$$\binom{n}{k} = \binom{k-1-n}{k}(-1)^{k} \ . \tag{2}$$

If n is negative, but close to 0, then the factor n in the right hand side of (1) keeps accuracy on the contrary to the expression $k-1-n$. In (2), n occurrs only in $k-1-n$, from which the number $-n$ is later reconstructed as $(k-1-n)-(k-1)$. This introduces a larger relative error compared to a direct use of n.

The difference between (1) and (2) is most remarkable for $-10^{-7} \ll n < 0$. In this case, the original implementation produces 0. Due to this, the difference may be demonstrated only for $n \le -10^{-7}$, where it is not large. For example

```
n <-  - 1.04e-7
x <- choose(n, 30)
y <- prod((n - 0:29)/(1:30))
(x - y)/y # [1] 1.441896e-08
```

With patch C, we get

```
(x - y)/y # [1] 4.772194e-15
```

Transformation 2

If the product $n(n-1)\dots(n-k+1)$ contains both positive and negative factors, then patch C defines $l = round(n)$ and uses the following transformation, which is valid for every $0 \le l \le k-1$.

$$\binom{n}{k} = \frac{\binom{n}{l}(n-l)\binom{n-l-1}{k-l-1}}{\binom{k}{l}(k-l)} = \frac{\binom{n}{l}(n-l)\binom{k-n-1}{k-l-1}(-1)^{k-l-1}}{\binom{k}{l}(k-l)} \tag{3}$$

The three binomial coefficients in the right hand side of (3) are then evaluated using `lbeta()` as $B(n-l+1, l+1)$, $B(l-n+1, k-l)$, $B(k-l+1, l+1)$. In these calls, the first argument of $B(,)$ is always at least 0.5 and the second is at least 1. In this situation, the function $lbeta()$ is numerically stable.

The original implementation uses in this case Gamma function (`lgammafn_sign()`) instead of Beta, which is less stable than (3) and sometimes produces warnings concerning the accuracy. For example

```
choose(19 - 2e-7, 30)
# [1] -3.328339e-16
# Warning message:
# In choose(19 - 2e-07, 30) : full precision was not achieved in 'lgamma'
```