Contents

Preface XI

1 Matrices
M. Fiedler 1
1.1 Basic notions on matrices, determinants 1
1.2 Norms, basic numerical linear algebra 14
1.3 Symmetric matrices 19
1.4 Generalized inverses 23
1.5 Nonnegative matrices, M- and P-matrices 27
1.6 Examples of other special classes of matrices 31

2 Solvability of systems of interval linear equations and inequalities
J. Rohn 35
2.1 Introduction and notations 35
2.2 An algorithm for generating Y_m 36
2.3 Auxiliary complexity result 38
2.4 Solvability and feasibility 40
2.5 Interval matrices and vectors 43
2.6 Weak and strong solvability/feasibility 45
2.7 Weak solvability of equations 47
2.8 Weak feasibility of equations 49
2.9 Strong solvability of equations 50
2.10 Strong feasibility of equations 55
2.11 Weak solvability of inequalities 57
2.12 Weak feasibility of inequalities 58
2.13 Strong solvability of inequalities 59
2.14 Strong feasibility of inequalities 61
2.15 Summary I: Complexity results 61
2.16 Tolerance solutions 62
2.17 Control solutions 64

2.18 Algebraic solutions 65
2.19 The square case 66
2.20 Summary II: Solution types 74
2.21 Notes and references 74

3 Interval linear programming
J. Rohn 79
3.1 Linear programming: Duality 79
3.2 Interval linear programming problem 83
3.3 Range of the optimal value 84
3.4 The lower bound 87
3.5 The upper bound 92
3.6 Finite range 96
3.7 An algorithm for computing the range 98
3.8 Notes and references 98

4 Linear programming with set coefficients
J. Nedoma and J. Ramík 101
4.1 Introduction 101
4.2 LP with set coefficients 101
4.2.1 Strict, weak and strong feasibility 102
4.2.2 Objective function 104
4.3 Duality 105
4.3.1 Weak duality 106
4.3.2 Strong duality 108
4.4 Generalized simplex method 111
4.4.1 Finding the dual optimal solution: Algorithm 112
4.4.2 Rectangular matrix 114
4.4.3 Finding the best basis: Algorithm 115
4.5 Conclusion 115

5 Fuzzy linear optimization
J. Ramík 117
5.1 Introduction 117
5.2 Fuzzy sets, fuzzy quantities 119
5.3 Fuzzy relations 122
5.4 Fuzzy linear optimization problems 127
5.5 Feasible solution 130
5.6 "Optimal" solution 134
5.6.1 Satisficing solution 134
5.6.2 α-efficient solution 138
5.7 Duality 140
5.8 Extended addition 145
5.9 Special models of FLP 149
5.9.1 Flexible linear programming 149

5.9.2 Interval linear programming 151
5.9.3 FLP problems with centered parameters 153
5.10 Fuzzy multicriteria linear programming problem 156
5.11 Numerical example 160
5.12 Conclusion 164

6 Interval linear systems and optimization problems over max-algebras
K. Zimmermann 165
6.1 Introduction 165
6.2 Max-separable functions and max-separable optimization problems 165
6.3 Extremal algebra notation 175
6.4 Noninterval systems of $(\oplus, \otimes)$-linear equations and inequalities 178
6.5 Noninterval max-separable optimization problems with $(\oplus, \otimes)$-linear constraints 182
6.6 $(\oplus, \otimes)$-linear systems of equalities and inequalities with interval coefficients 185
6.7 Optimization problems with $(\oplus, \otimes)$-linear interval constraints . 190
6.8 Conclusion 193

References 195

List of Symbols 205

Index 209