Asymptotic Properties of Large Graphs

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Martin Doležal, Jan Grebík, JH, Israel Rocha, Václav Rozhoň:
Cut distance identifying graphon parameters over weak limits*

Structure of graphs: information-theoretic perspective

Szemerédi’s regularity lemma 1978

For every ε>0 there exists an M(ε) so that each graph can be ε-approximately represented by a matrix of “densities” of dimensions M(ε)×M(ε).

Szemerédi’s theorem about arithmetic progressions 1975
Abel prize 2012

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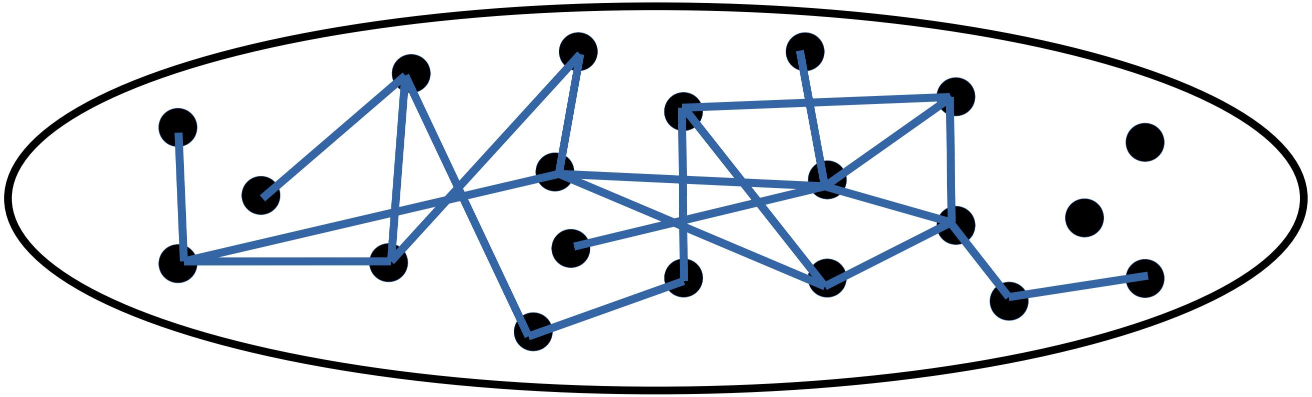

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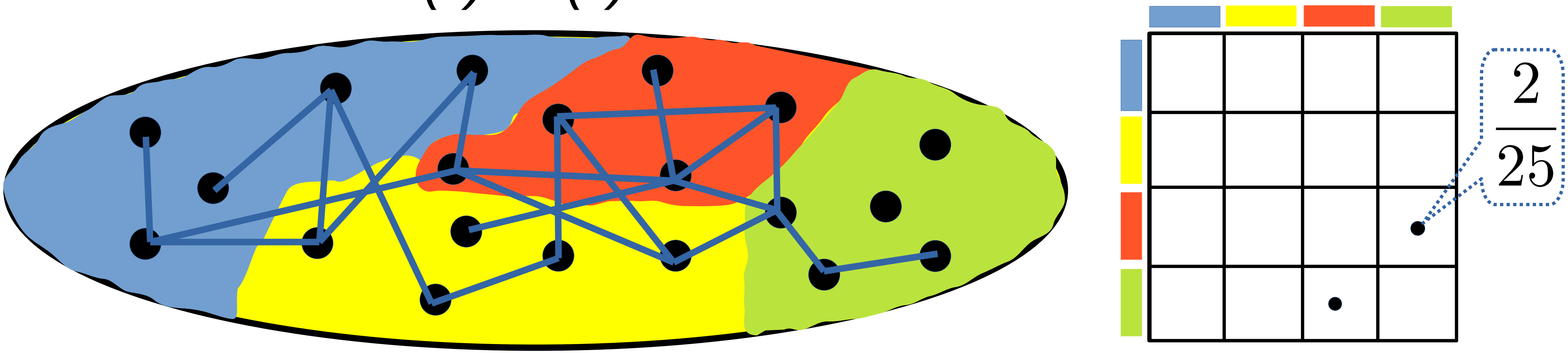

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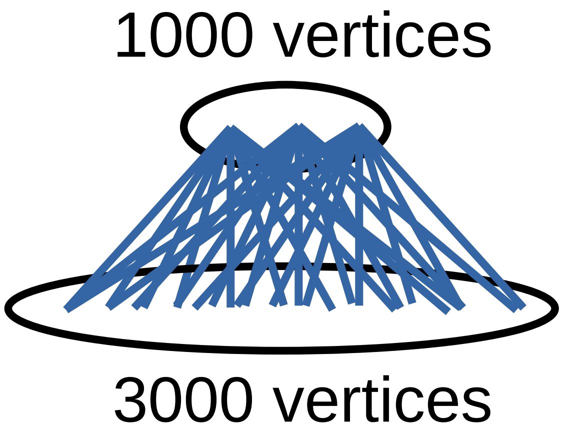

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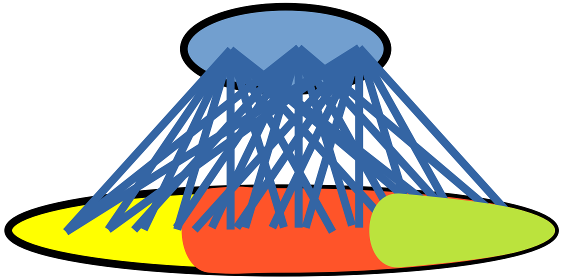

0	1	1	1
1	0	0	0
1	0	0	0
1	0	0	0

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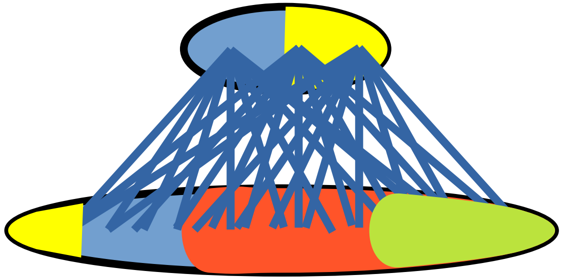

.5	.5	.5	.5
.5	.5	.5	.5
.5	.5	0	0
.5	.5	0	0

Graphons (a.k.a. graph limits)

Lovász, Szegedy 2004

Fulkerson Prize 2012

Borgs, Chayes, Lovász, Sós, Vesztergombi, ...

- **Main idea:** Compactify the space of finite graphs.
- **Definition:** A graphon is a symmetric measurable function

$$W:[0,1]^2 \to [0,1]$$

- **Why would you want that?**

Compactness of graphons

For every sequence G_1, G_2, … of graphs there exists a graphon W and a subsequence G_{i1}, G_{i2} converging to it.

Lovász—Szegedy ‘04, *Szemerédi’s regularity lemma*

Diaconis—Janson ‘08, *exchangeable arrays (Aldous–Hoover, 1970’s)*

Elek—Szegedy ‘12, *nonstandard analysis*

Doležal—H. ‘19, Doležal—Grebík—H.—Rocha—Rozhoň ‘21, ‘22

weak topology*