Algebras from a Quasitopos of Rough Sets

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Outline

- Rough sets
- Categories of rough sets
- Generalization of categories of rough sets
- Algebra over subobjects of a rough set
- c.c.-pseudo-Boolean algebras

Rough Sets (Pawlak, Z. (1982))

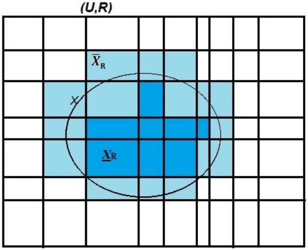
- Rough set theory was first proposed to deal with incomplete information systems and vagueness.

- (U, R), with U a set and R an equivalence relation over U, is called a Pawlak approximation space. For a subset $X \subseteq U$, consider
$$\overline{X}_R := \{x \mid [x]_R \cap X \neq \emptyset\}, \text{ and}$$
$$\underline{X}_R := \{x \mid [x]_R \subseteq X\}$$
where $[x]_R$ is an equivalence class in U containing x.

- $\overline{X}_R$ is called R-upper approximation of X and $\underline{X}_R$ is called R-lower approximation of X.

Rough sets

- The pair $(\underline{X}_R, \overline{X}_R)$ is called a rough set in the approximation space (U, R). Here, $\underline{X}_R \subseteq \overline{X}_R$.



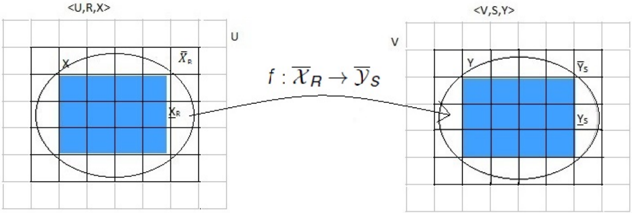
- Let $\overline{\mathcal{X}}_R$ and $\underline{\mathcal{X}}_R$ denote the collections of equivalence classes of R contained in $\overline{X}_R$ and $\underline{X}_R$ respectively, that is,

$$\overline{X}_R = \bigcup \overline{\mathcal{X}}_R \qquad \text{and} \qquad \underline{X}_R = \bigcup \underline{\mathcal{X}}_R.$$

ROUGH Category (Banerjee, M. and Chakraborty, M.K. (1993))

- Objects of *ROUGH* have the form (U, R, X), where U is a set, R an equivalence relation on U and X a subset of U.

- An arrow in *ROUGH* with *dom* (U, R, X) and *cod* (V, S, Y) is a map $f : \overline{\mathcal{X}}_R \to \overline{\mathcal{Y}}_S$ such that $f(\underline{\mathcal{X}}_R) \subseteq \underline{\mathcal{Y}}_S$.

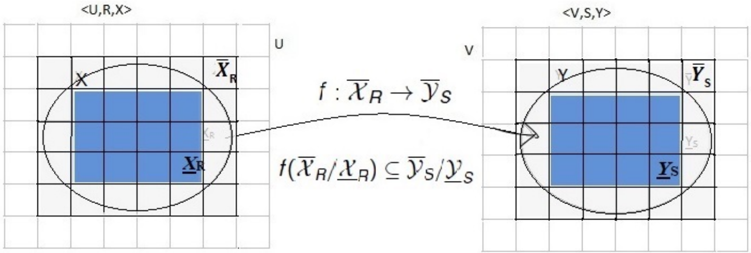


- Note that the lower approximation is preserved by the arrow f.

- *ROUGH* is not a topos.

ξ-*ROUGH* Category (Banerjee, M. and Chakraborty, M.K. (1993))

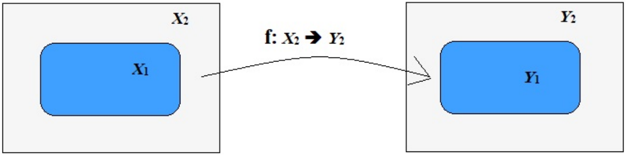
- Objects of ξ-*ROUGH* are same as of *ROUGH*.
- A ξ-*ROUGH* arrow f is a *ROUGH* arrow with *dom* (U, R, X) and *cod* (V, S, Y) such that $f(\overline{\mathcal{X}}_R/\underline{\mathcal{X}}_R) \subseteq \overline{\mathcal{Y}}_S/\underline{\mathcal{Y}}_S$.



- Note that the lower approximation $\underline{\mathcal{X}}_R$, as well as the boundary region $\overline{\mathcal{X}}_R/\underline{\mathcal{X}}_R$, is preserved by the arrow f.

RSC Category (Li, X.S., Yuan, X.H. (2008))

- Objects of *RSC* are (X_1, X_2) where X_1, X_2 are sets and $X_1 \subseteq X_2$.
- An *RSC* arrow with *dom* (X_1, X_2) and *cod* (Y_1, Y_2) is a map $f : X_2 \to Y_2$ such that $f(X_1) \subseteq Y_1$.



Comparison

	ROUGH	ξ-*ROUGH*	RSC (Li, Yuan (2008))
Objects	(U, R, X)	(U, R, X)	(X_1, X_2)
Morphisms	$f : \overline{\mathcal{X}}_R \to \overline{\mathcal{Y}}_S$ $f(\underline{\mathcal{X}}_R) \subseteq \underline{\mathcal{Y}}_S$	$f : \overline{\mathcal{X}}_R \to \overline{\mathcal{Y}}_S$ $f(\underline{\mathcal{X}}_R) \subseteq \underline{\mathcal{Y}}_S$ $f(\overline{\mathcal{X}}_R \setminus \underline{\mathcal{X}}_R) \subseteq \overline{\mathcal{Y}}_S \setminus \underline{\mathcal{Y}}_S$	$f : X_2 \to Y_2$ $f(X_1) \subseteq Y_1$

Theorem (More, A. K. and Banerjee, M.)

1. *ROUGH is equivalent to RSC, and forms a quasitopos.*
2. ξ*-ROUGH is equivalent to SET*^{2}*, and forms a topos.*

Subobjects and strong subobjects in *RSC*

- In figure (a), (X_1, X_2) is a *subobject* of (Y_1, Y_2).
- In figure (b), (X_1, X_2) is a *strong subobject* of (Y_1, Y_2).

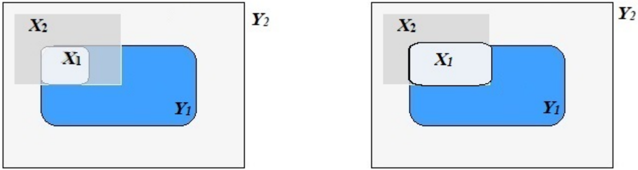


Figure (a) $X_1 \neq X_2 \cap Y_1$ Figure (b) $X_1 = X_2 \cap Y_1$

- Hereafter, the strong subobejcts of *RSC* are referred as the subobjects of *RSC*.

The category *RSC* is based on sets. Let us replace sets by an arbitrary topos $\mathscr{C}$.

- $RSC(\mathscr{C})$ category: Objects are pairs (A, B) where A and B are objects in $\mathscr{C}$ such that there exist a monic arrow $m : A \to B$ in $\mathscr{C}$.

  An arrow with *dom* (X_1, X_2) and *cod* (Y_1, Y_2) is a pair of arrows (f', f) in $\mathscr{C}$, such that the following diagram commutes in $\mathscr{C}$.

$$\begin{tikzcd} X_1 \arrow[r, "f'"] \arrow[d, "m"'] & Y_1 \arrow[d, "m'"] \\ X_2 \arrow[r, "f"'] & Y_2 \end{tikzcd}$$

  where m and m' are monic arrows corresponding to the objects (X_1, X_2) and (Y_1, Y_2) in $RSC(\mathscr{C})$.

$RSC(\mathscr{C})$

Theorem

$RSC(\mathscr{C})$ is a quasitopos.

- $RSC(SET)$ is the category RSC.
- On generalizing RSC to $RSC(\mathscr{C})$, we have lost the Boolean property $a \vee \neg a = 1$ in the algebra of subobjects of the quasitopos RSC.

An example of $RSC(\mathscr{C})$

- Consider $\mathscr{C}$ to be topos **M-Set**, where **M** is a monoid.
- An object of **M-Set** is a monoid action on a set X, and an arrow is a function preserving monoid action.
- **M-Set** is not a Boolean topos, when **M** is not a group.
- Consider **M** to be $\mathbf{2} = \{0, 1\}$ with $0 \leq 1$. What are the objects and arrows of $RSC(\textbf{2-Set})$?

RSC(**2-Set**)

- An object is a triple (X_1, X_2, μ) such that $X_1 \subseteq X_2$ and $\mu : X_2 \to Y_2$ is a set function such that $\mu^2 = \mu$ and $\mu|_{X_1} : X_1 \to X_1$.
- An arrow $f : (X_1, X_2, \mu) \to (Y_1, Y_2, \lambda)$ is the set function $f : X_2 \to Y_2$ such that $f(X_1) \subseteq Y_1$ and $\lambda f = f\mu$.

$$\begin{tikzcd} X_2 \arrow[r, "f"] \arrow[d, "\mu"'] & Y_2 \arrow[d, "\lambda"] \\ X_2 \arrow[r, "f"'] & Y_2 \end{tikzcd}$$

- RSC(**2-Set**) gives the motivation of defining *monoid action on rough sets*.

- A monoid $\mathbf{M} = (M, *, e)$ action on a set X is a function $\lambda : M \times X \to X$ satisfying

$$\lambda(e, x) = x \quad \text{and}$$

$$\lambda(m, \lambda(p, x)) = \lambda(m * p, x).$$

Definition (Monoid Action on rough sets)

A monoid $\mathbf{M}$ action on a rough set (X_1, X_2) is a triple (X_1, X_2, μ) such that $\mu : \mathbf{M} \times X_2 \to X_2$ is a monoid action of $\mathbf{M}$ on the set X_2, with the condition that $\mu|_{X_1}$ is a monoid action of $\mathbf{M}$ on X_1.

Algebra of subobjects of an *RSC* object

- Any topos (quasitopos) has an intuitionistic logic associated with the (strong) subobjects of its objects.
- Let $\mathcal{M}$ be the collection of subobjects of an *RSC*-object (U_1, U_2), that is,

$$\mathcal{M} = \{(A_1, A_2) \mid A_1 \subseteq U_1, \;\; A_2 \subseteq U_2, \;\; A_1 = U_1 \cap A_2\}.$$

- Propositional Connectives are obtained as following:

$$\cap : \; (A_1, A_2) \cap (B_1, B_2) = (A_1 \cap B_1, A_2 \cap B_2)$$

$$\cup : \; (A_1, A_2) \cup (B_1, B_2) = (A_1 \cup B_1, A_2 \cup B_2)$$

$$\neg : \; \neg(A_1, A_2) = (U_1 \setminus A_1, U_2 \setminus A_2)$$

$$\rightarrow: \; (A_1, A_2) \rightarrow (B_1, B_2) = (U_1 \setminus A_1, U_2 \setminus A_2) \cup (B_1, B_2)$$

Observations

- The algebraic structure of subobjects of an object in quasitopos *ROUGH* is same as that of topos ξ-*ROUGH*.
- The algebra obtained is Boolean, and thus the corresponding logic obtained is classical.
- On a close look at negation $\neg$, we see that negation is with respect to fixed *RSC*-object (U_1, U_2).
- Therefore, we need to use the notion of relative negation in rough sets.

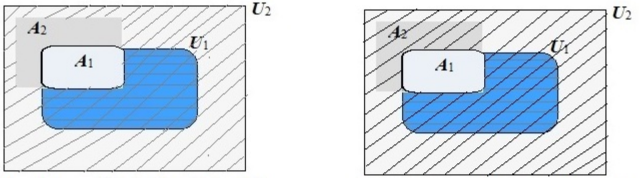
Complementation in Rough Sets

- Relative rough complement, defined by Iwiński (1987), in Rough sets is given by

$$(A_1, A_2) - (B_1, B_2) = (A_1 \setminus B_2, A_2 \setminus B_1)$$

- In the lines of this, we define the negation as

$$\sim : \sim (A_1, A_2) := (U_1 \setminus A_2, U_2 \setminus A_1).$$



$\neg(A_1, A_2) = (U_1 \setminus A_1, U_2 \setminus A_2)$ $\sim (A_1, A_2) := (U_1 \setminus A_2, U_2 \setminus A_1)$

- This 'results' in a different algebraic structure on $\mathcal{M}$, namely *c.* $\vee$ *c.* lattices.

Some properties of negation $\sim$

1. $\sim (\underline{\mathcal{U}}, \overline{\mathcal{U}}) = (\emptyset, \overline{\mathcal{U}} \setminus \underline{\mathcal{U}})$ $\qquad \sim 1 \neq 0$
2. $\sim (\emptyset, \emptyset) = (\underline{\mathcal{U}}, \overline{\mathcal{U}})$ $\qquad \sim 0 = 1$
3. $\sim\sim (A_1, A_2) = (A_1, A_2 \cup (\overline{\mathcal{U}} \setminus \underline{\mathcal{U}}))$ $\qquad \sim\sim a \neq a$
4. $\sim\sim\sim (A_1, A_2) = \sim (A_1, A_2)$ $\qquad \sim\sim\sim a = \sim a$
5. $(A_1, A_2) \cup \sim (A_1, A_2) = (\underline{\mathcal{U}}, \overline{\mathcal{U}})$ $\qquad a \vee \sim a = 1$
6. $(A_1, A_2) \cap \sim (A_1, A_2) = (\emptyset, A_2 \setminus A_1)$ $\qquad a \wedge \sim a \neq 0$
7. DeMorgan's laws hold.

Definition (C.C. lattice)

A contrapositionally complemented ($c.c.$) lattice is an algebra of the form $(B, \vee, \wedge, \rightarrow, \neg, 1)$ such that the reduct $(B, \vee, \wedge, \rightarrow, 1)$ is a relatively pseudo-complemented ($r.p.c.$) lattice and $\neg$ satisfies the contraposition law

$$x \rightarrow \neg y = y \rightarrow \neg x.$$

equivalently, $\neg a = a \rightarrow \neg 1$.

The logic corresponding to the class of $c.c.$ lattices is the minimal logic.

Definition ($C. \vee C.$ Lattices)

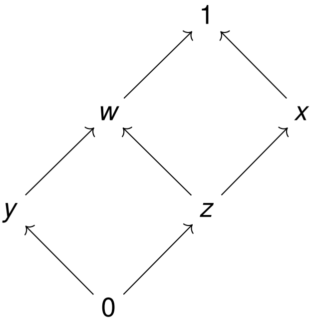
A contrapositionally $\vee$ complemented ($c. \vee c.$) lattice is a $c.c.$ lattice satisfying

$$x \vee \neg x = 1.$$

Examples

- Let us consider the following 6-element *r.p.c.* lattice A.

a	$\neg a$ $a \to 0$	$\neg_1 a$ $a \to w$	$\neg_2 a$ $a \to x$	$\neg_3 a$ $a \to y$
0	1	1	1	1
y	x	1	x	1
z	y	1	1	y
w	0	1	x	y
x	y	w	1	y
1	0	w	x	y
	Heyting	$c. \vee c.$	$c. \vee c.$	



- A does not form $c. \vee c.$ lattice with the negation $\neg_3$.

$C. \vee C.$ lattices from a Boolean Algebra

Consider a Boolean Algebra $\mathscr{B} = (B, \leq, \vee, \wedge, \neg, \rightarrow, 0, 1)$ and $u = (u_1, u_2)$ where $u_1, u_2 \in B$.

Consider the set $A_u = \{(a_1, a_2) : a_1 \leq a_2, \;\; a_1, a_2 \in B, \;\; a_1 = a_2 \wedge u_1\}$.

Define the following operations on A_u:

- $(a_1, a_2) \vee (b_1, b_2) := (a_1 \vee b_1, a_2 \vee b_2)$
- $(a_1, a_2) \wedge (b_1, b_2) := (a_1 \wedge b_1, a_2 \wedge b_2)$
- $\sim (a_1, a_2) := (u_1 \wedge \neg a_2, u_2 \wedge \neg a_1)$
- $(a_1, a_2) \rightarrow (b_1, b_2) := (u_1 \wedge \neg a_1, u_2 \wedge \neg a_2) \vee (b_1, b_2)$

$\mathscr{A}_u := (A_u, \vee, \wedge, \rightarrow, \sim, 0, 1)$ forms a $C. \vee C.$ lattice with the least element 0.

Algebra of Subobjects of a $RSC(\mathscr{C})$ object

- Let $\mathcal{M}((U_1, U_2))$ denote the set of strong monics of an $RSC(\mathscr{C})$-object (U_1, U_2).

- $\mathcal{M}((U_1, U_2))$ forms a pseudo-Boolean (Heyting) algebra with propositional connectives as follows.

$$\cap : \ (f', f) \cap (g', g) = (f' \cap g', f \cap g)$$

$$\cup : \ (f', f) \cup (g', g) = (f' \cup g', f \cup g)$$

$$\neg : \ \neg(f', f) = (\neg f', \neg f)$$

$$\rightarrow : \ (f', f) \rightarrow (g', g) = (f' \rightarrow g', f \rightarrow g)$$

where $(f', f), (g', g) \in \mathcal{M}((U_1, U_2))$.

- We have observed that in RSC, $\mathcal{M}((U_1, U_2))$ forms a Boolean algebra.

Complementation in Rough Sets

- In the lines of the negation defined by Iwiński on rough sets, we give the new definition of negation for $RSC(\mathscr{C})$, as done for RSC,

$$\sim: \sim (f', f) := (\neg f', \neg(m \circ f')).$$

where $(f', f) \in \mathcal{M}((U_1, U_2))$ and $m : U_1 \to U_2$ is a monic arrow corresponding to (U_1, U_2).

- $\sim$ satisfies the contraposition law, but is neither a semi-negation nor involutive.

$$\sim (a \to a) \to b \neq 1$$

$$\sim\sim a \neq a$$

- We also have $\sim (Id_{U_1}, Id_{U_2}) = \neg\neg \sim (Id_{U_1}, Id_{U_2})$.
- This results in a new algebraic structure on $\mathcal{M}((U_1, U_2))$, namely *Contrapositionally complemented pseudo-Boolean algebras.*

Contrapositionally complemented pseudo-Boolean algebras

Definition ($c.c.$-pseudo-Boolean algebra)

An abstract algebra $\mathcal{A} := (A, 1, 0, \to, \cup, \cap, \neg, \sim)$ is said to be a $c.c.$-pseudo-Boolean algebra if $(A, 1, 0, \to, \cup, \cap, \neg)$ forms a pseudo-Boolean algebra and satisfies

$$\sim a = a \to (\neg\neg \sim 1)$$

for all $a \in A$.

If, in addition, $x \vee \sim x = 1$ for all $x \in A$, $\mathcal{A}$ forms a $c. \vee c.$-pseudo-Boolean algebra.

- The reduct $(A, 1, 0, \to, \cup, \cap, \sim)$ forms a $c.c.$ lattice with the least element 0.

Some properties of negation $\sim$ in *c.c.*-pseudo-Boolean algebras

1. $\sim 1 = \neg\neg \sim 1$.
2. $\sim 0 = 1$.
3. $\neg\neg \sim x = \sim x$.
4. $\sim\sim\sim x = \sim x$.
5. $\neg x \leq \sim x$.

These are also true for $c. \vee c.$-pseudo-Boolean algebras.

Some properties of negation $\sim$ in $c. \vee c.$-pseudo-Boolean algebras

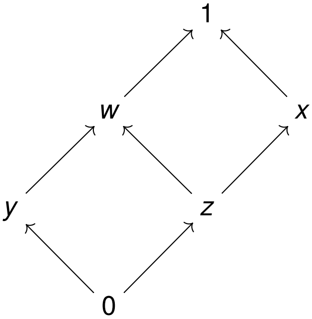
1. $x \vee \sim x = 1.$
2. $\sim (x \wedge y) = \sim x \vee \sim y.$
3. $\neg \sim x \leq x.$

These are NOT true for $c.c.$-pseudo-Boolean algebras.

Examples

- Let us again see the 6-element *r.p.c.* lattice A discussed previously.

a	$\neg a$	$\sim_1 a$ $a \to w$	$\sim_2 a$ $a \to x$	$\sim_3 a$ $a \to y$
0	1	1	1	1
y	x	1	x	1
z	y	1	1	y
w	0	1	x	y
x	y	w	1	y
1	0	w	x	y
			$c. \vee c.$-p-B	$c.c.$-p-B



- $(A, 1, 0, \to, \cup, \cap, \neg, \sim_1)$ neither forms $c.c.$-pseudo-Boolean nor $c. \vee c.$-pseudo-Boolean algebra.

Examples

- $(\mathcal{M}((U_1, U_2)), (U_1, U_2), (0, 0), \cap, \cup, \rightarrow, \neg, \sim)$ forms a *c*.*c*.-pseudo-Boolean algebra, for each $RSC(\mathscr{C})$-object (U_1, U_2).

- $(\mathcal{M}(X), (\underline{\mathcal{X}}, \overline{\mathcal{X}}), \cap, \cup, \rightarrow, \neg, \sim)$ forms a *c*. $\vee$ *c*.-pseudo-Boolean algebra, for each *RSC*-object $(\underline{\mathcal{X}}, \overline{\mathcal{X}})$.

- An entire class of *c*.*c*.-pseudo-Boolean algebras can be obtained starting from any arbitrary pseudo-Boolean algebra $\mathcal{H} := (H, 1, 0, \rightarrow, \cup, \cap, \neg)$.

Examples

- Let $\mathcal{H}^{[2]} := \{(a,b) : \ a \leq b, \ a, b \in H\}$, $u := (u_1, u_2) \in \mathcal{H}^{[2]}$, and $A_u := \{(a_1, a_2) \in \mathcal{H}^{[2]} \ : a_2 \leq u_2 \text{ and } \ a_1 = a_2 \wedge u_1\}$

- Define the following operators on A_u:

$$\sqcup : \ (a_1, a_2) \sqcup (b_1, b_2) := (a_1 \vee b_1, a_2 \vee b_2)$$

$$\sqcap : \ (a_1, a_2) \sqcap (b_1, b_2) := (a_1 \wedge b_1, a_2 \wedge b_2)$$

$$\neg : \ \neg(a_1, a_2) := (u_1 \wedge \neg a_1, u_2 \wedge \neg a_2)$$

$$\sim : \sim (a_1, a_2) := (u_1 \wedge \neg a_1, u_2 \wedge \neg a_1)$$

$$\rightarrow : \ (a_1, a_2) \rightarrow (b_1, b_2) := ((a_1 \rightarrow b_1) \wedge u_1, (a_2 \rightarrow b_2) \wedge u_2)$$

Examples

Proposition

$\mathcal{A}_u := (A_u, u, (0,0), \rightarrow, \sqcup, \sqcap, \neg, \sim)$ *is a c.c.-pseudo-Boolean algebra.*

- If $\mathcal{H}$ is Boolean, we have $a \vee \sim a = 1$ for any $u = (u_1, u_2)$.

Proposition

If $\mathcal{H}$ is Boolean, then $\mathcal{A}_u$ forms a c. $\vee$ c.-pseudo-Boolean algebra.

Representation Theorem for *c.c.*-pseudo-Boolean algebras

Definition (Contrapositionally complemented pseudo-fields)

Let $\mathscr{G}(X) := (\mathcal{G}(X), X, \emptyset, \cap, \cup, \rightarrow, \neg)$ be a pseudo-field of open subsets of a topological space X. Define

$$\begin{aligned} \sim X &:= \neg\neg Y_0 \quad \text{for some } Y_0 \text{ belonging to } \mathcal{G}(X), \\ \sim Z &:= Z \rightarrow (\neg\neg \sim X). \end{aligned}$$

The algebra $(\mathcal{G}(X), X, \emptyset, \cap, \cup, \rightarrow, \neg, \sim)$ is called the contrapositionally complemented pseudo-field (*c.c.* pseudo-field) of open subsets of X.

Theorem (Representation Theorem)

Let $\mathcal{A} := (A, 1, 0, \rightarrow, \cup, \cap, \neg, \sim)$ be a c.c.-pseudo-Boolean algebra. There exists a monomorphism h from $\mathcal{A}$ into a c.c.-pseudo-field of all open subsets of a topological space X.

Properties of *c.c.*-pseudo-Boolean algebras

- Since the class of all pseudo-Boolean algebras is equationally definable, the class of all c.c.-pseudo-Boolean algebras is also so.
- The logic corresponding to *c.c.*-pseudo-Boolean algebras can be defined. We call it *Intuitionistic logic with minimal negation* (ILM).
- Intuitionistic logic (IL) is embedded inside ILM.
- A natural question is whether some 'interpretation' of ILM in IL exists?

Interpretation

Various definitions of mappings from one formal system to another can be found in literature (Eg. Prawitz and Malmnäs).

Let us define a general 'interpretation' between two mappings.

Definition (Interpretation)

Consider two formal logics $\mathfrak{L}_1$ and $\mathfrak{L}_2$. The mapping $r : L_1 \rightarrow L_2$, from the set L_1 of formulas in $\mathfrak{L}_1$ to the set L_2 of formulas in $\mathfrak{L}_2$, is called an interpretation of $\mathfrak{L}_1$ in $\mathfrak{L}_2$, if for any formula $\alpha \in L_1$, we have the following condition:

$$\vdash_{\mathfrak{L}_1} \alpha \text{ if and only if } \Delta_\alpha \vdash_{\mathfrak{L}_2} r(\alpha),$$

where Δ_α is a finite set of formulas in $\mathfrak{L}_2$ corresponding to α.

Interpretation from ILM into IL

Theorem

There exists an interpretation from ILM onto IL, that is, the mapping $r : ILM \to IL$ is onto.

Proof.

For any ILM-formula α with $p_1, \ldots, p_n$ propositional variables occurring in α, there exists a ILM-formula α^* such that

1. α^* does not contain $\sim$,
2. α^* contains $p_1, \ldots, p_n$ and a distinct propositional variable q,
3. and if $\vdash_{\mathrm{ILM}} \sim \top \leftrightarrow q$, then $\vdash_{\mathrm{ILM}} \alpha \leftrightarrow \alpha^*$.

Define $r(\alpha) = \alpha^*$ and $\Delta_\alpha = \{\neg\neg q \to q\}$. We obtain the following:

$$\vdash_{\mathrm{ILM}} \alpha \leftrightarrow \{\beta\} \vdash_{\mathrm{IL}} \alpha^*$$

□

Conclusions and future work

- *ROUGH* and ξ-*ROUGH* are based on preserving some ‘regions’ of an approximation space. There can be other possbile categories of rough sets based upon conditions on different ‘regions’.
- Monoid actions on rough sets seems promising area, as monoid actions have wide-ranging applications from linguistics to morphology.
- We saw the representation theorem for $c.c$-pseudo-Boolean algebra. Further, the representation theorem of $c. \vee c.$-pseudo-Boolean algebra has to be found.
- Other semantics of the logic ILM has to be looked upon, mainly based on the Dunn’s kite diagram of negations.

References

1. More, A.K., Banerjee, M.: Categories and algebras from rough sets: New facets. Fundamenta Informaticae 148(1-2) (2016) 173-190.
2. More, A.K., Banerjee, M.: New Algebras and Logic from a Category of Rough Sets, IJCRS 2017, Part I, LNAI 10313 (To appear).
3. Pawlak, Z.: Rough sets. International Journal of Computer and Information Sciences 11(5) (1982) 341-356.
4. Banerjee, M., Chakraborty, M.K.: A category for rough sets. Found. of Comput. and Deci. Sci. 18(3-4) (1993) 167-180.
5. Iwiński, T.B.: Algebraic approach to rough sets. Bull. Polish Acad. Sci. Math. 35 (1987) 673-683.

References

1. Li, X.S., Yuan, X.H.: The category RSC of I-rough sets. In: 5th ICFSKD, 2008. Volume 1. (Oct 2008) 448-452.

2. Banerjee, M., Chakraborty, M.K.: Algebras from rough sets. In Pal, S.K., Polkowski, L., Skowron, A., eds.: Rough-Neural Computing. Cognitive Technologies. (2004) 157-184.

3. Prawitz, D., Malmnäs, P.E.: A survey of some connections between classical, intuitionistic and minimal logic. Studies in Logic and the Foundations of Mathematics 50 (1968) 215-229.

4. Rasiowa, H.: An Algebraic Approach to Non-classical Logics. Studies in Logic and the Foundations of Mathematics.

Thank you.