

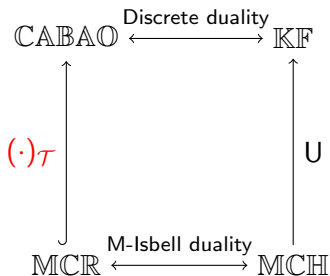
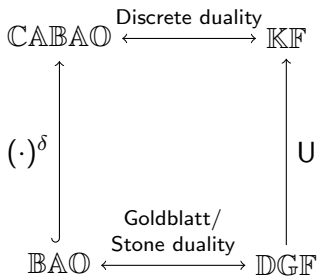
Algorithmic Sahlqvist Preservation for Modal Compact Hausdorff Spaces

Zhiguang Zhao

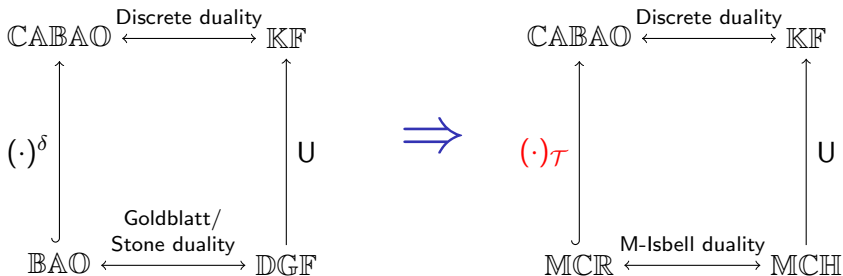
Delft University of Technology, Delft, the Netherlands

TACL, Prague, 28th Jun, 2017

Motivation and Aim



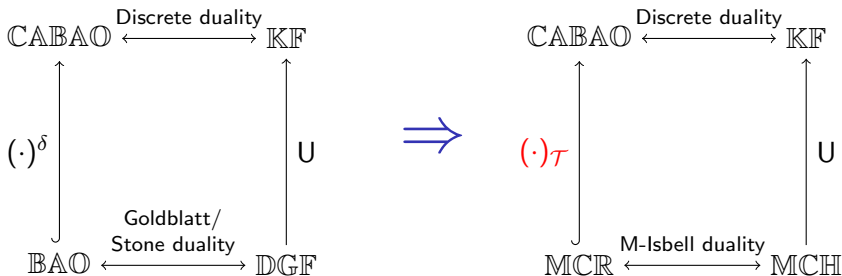
Motivation and Aim



Which inequalities are preserved by the embedding $(\cdot)_T$?

G. Bezhanishvili, N. Bezhanishvili, and J. Harding. "Modal compact Hausdorff spaces". *JLC*, 25(1):1–35, 2015.

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We explore this question with the **ALBA** technology

The ALBA technology

- Calculus/Algorithm which computes the first-order correspondent of input formulas/inequalities;
- Based on the order-theoretic properties of the algebraic interpretation of logical connectives;
- Guaranteed to succeed on **inductive formulas/inequalities**;
- Inductive formulas/inequalities generalise Sahlqvist formulas/inequalities;
- General definition for arbitrary normal and regular LEs, based on the order-theoretic properties of the algebraic interpretation of logical connectives.

Standard proof-strategy for canonicity via ALBA

$$\begin{array}{ccc} \mathbb{A} \models \alpha \leq \beta & & \mathbb{A}^\delta \models \alpha \leq \beta \\ \Downarrow & & \Downarrow \\ \mathbb{A}^\delta \models_{\mathbb{A}} \alpha \leq \beta & & \\ \Downarrow & & \\ \mathbb{A}^\delta \models_{\mathbb{A}} \text{ALBA}(\alpha \leq \beta) & \iff & \mathbb{A}^\delta \models \text{ALBA}(\alpha \leq \beta) \end{array}$$

Let us generalise this strategy to modal compact Hausdorff spaces.

U-shaped Argument

\mathcal{T} : modal compact Hausdorff space;

$\mathbb{L}_{\mathcal{T}}$: the modal compact regular frame associated with \mathcal{T} ;

$\mathbb{F}_{\mathcal{T}}$: the underlying Kripke frame of \mathcal{T} ;

$\mathbb{B}_{\mathbb{F}_{\mathcal{T}}}$: the complex algebra of $\mathbb{F}_{\mathcal{T}}$.

$$\begin{array}{ccc} \mathbb{L}_{\mathcal{T}} \models \phi \leq \psi & & \mathbb{B}_{\mathbb{F}_{\mathcal{T}}} \models \phi \leq \psi \\ \Downarrow & & \\ \mathbb{B}_{\mathbb{F}_{\mathcal{T}}} \models_{\mathbb{L}_{\mathcal{T}}} \phi \leq \psi & & \Downarrow \\ \mathbb{B}_{\mathbb{F}_{\mathcal{T}}} \models_{\mathbb{L}_{\mathcal{T}}} \text{Pure}(\phi \leq \psi) & \Leftrightarrow & \mathbb{B}_{\mathbb{F}_{\mathcal{T}}} \models \text{Pure}(\phi \leq \psi) \end{array}$$

To implement this strategy, we need to adapt ALBA to the modal compact Hausdorff setting.

Main Results

- ALBA adapted to the modal compact Hausdorff setting (**MH-ALBA**);
- Rules of MH-ALBA **sound** w.r.t. **open** assignments (i.e. the assignments mapping into $\mathbb{L}_{\mathcal{T}}$);
- MH-ALBA succeeds on **1-Sahlqvist inequalities** and **∂ -Sahlqvist inequalities**;
- Corollary: 1-Sahlqvist and ∂ -Sahlqvist inequalities are **Hausdorff-canonical**.

Main Results

Prop: set of propositional variables,

$$\mathcal{L} \ni \varphi ::= p \mid \perp \mid \top \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \Box \varphi \mid \Diamond \varphi.$$

Definition (1-Sahlqvist inequalities)

$\phi \leq \psi$ is 1-Sahlqvist if $\phi = \phi'(\chi_1/z_1, \dots, \chi_n/z_n)$ such that

1. $\phi'(z_1, \dots, z_n)$ is built out of \wedge, \vee, \Diamond ;
2. every χ is of the form $\Box^n p, \Box^n \top, \Box^n \perp$ for $n \geq 0$;
3. ψ is a formula in the positive modal language.

The ∂ -Sahlqvist inequality $\phi \leq \psi$ where $\psi = \psi'(\chi_1/z_1, \dots, \chi_n/z_n)$ is defined dually.

Future Work

- Extensions to arbitrary modal signatures and fixed-point operators;
- Extensions to inductive formulas;
- Studying the “Hausdorff-canonical extensions” of modal compact Hausdorff spaces purely algebraically.