Quasi injectivity of partially ordered acts

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TACL 26-30 June 2017
Charles University

Introduction

Actions of a Monoid on Sets

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Actions of a Monoid on Partially Ordered Sets

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Actions of a Monoid on Sets

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Actions of a Monoid on Partially Ordered Sets

$\Rightarrow$

Partially Ordered Sets with actions of a pomonoid S on them (Pos-S)

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Injectivity

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Banaschewski:

In the category of posets with order preserving maps (**Pos**)

$$\mathcal{E}\text{-injective} = \text{Complete posets}$$

Introduction

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Sikorski:

In the category of **BoolAlg**

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Introduction

Sikorski:

In the category of **BoolAlg**

$\mathcal{E}$-injective = Complete Boolean algebras

Ebrahimi, Mahmoudi, Rasouli:

- In **Pos**-S Mono-injective object = Trivial object
- **Pos**-S has enough $\mathcal{E}$-injective objects.

Quasi Injectivity

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- Ahsan, gave a characterization of monoids whose class of quasi injective S-acts is closed under the formation of direct sums. Also, he investigated monoids all of whose S-acts are quasi injective.
- Roueentan and Ershad, introduced duo S-act, inspired from the concept of duo module in module theory, which is tightly related to quasi injectivity.

Preliminaries

Definition (Act-S)

Recall that a right S-act or S-set for a monoid S is a set A equipped with an action $A \times S \to A$, $(a, s) \rightsquigarrow as$, such that
(1) $ae = a$ (e is the identity element of S),
(2) $a(st) = (as)t$, for all $a \in A$ and $s, t \in S$.

Let **Act-S** denote the category of all S-acts with action preserving maps ($f : A \to B$ with $f(as) = f(a)s$, for all $a \in A, s \in S$).

Preliminaries

Definition (Zero element)

Let A be an S-act. An element $a \in A$ is called a zero, fixed, or a trap element if $as = a$, for all $s \in S$.

Preliminaries

Definition (Po-monoid, Po-group)

A po-monoid (po-group) S is a monoid (group) with a partial order $\leq$ which is compatible with its binary operation (that is, for $s, t, s', t' \in S$, $s \leq t$ and $s' \leq t'$ imply $ss' \leq tt'$).

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Definition (Right (Left) Zero Semigroup)

A semigroup S is called *right (left) zero* if its multiplication is defined by $st = t(st = s)$, for all $s, t \in S$. Also, $S\dot{\cup}\{e\}$ is called a right (left) zero semigroup with an adjoined identity, if S is a right (left) zero semigroup and $se = s = es$, for all $s \in S$, and $ee = e$.

Preliminaries

Definition (Pos-S)

For a po-monoid S, a right S-poset is a poset A which is also an S-act whose action $\lambda : A \times S \to A$ is order-preserving, where $A \times S$ is considered as a poset with componentwise order. The category of all S-posets with action preserving monotone maps between them is denoted by **Pos-S**.

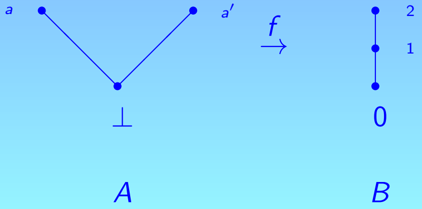
Definition (Order-embedding)

A morphism $f : A \to B$ in the category **Pos**-S and its subcategories, is called order-embedding, or briefly *embedding*, if for all $x, y \in A$, $f(x) \leq f(y)$ if and only if $x \leq y$.
We consider $\mathcal{E}$ to be the class of all embeddings in the category **Pos**-S and its subcategories.

Relation between Monomorphisms and Embeddings

In the category **Pos**-S :
Embedding $\Rightarrow$ Mono
Mono $\not\Rightarrow$ Embedding

Suppose that S is an arbitrary po-monoid. Define f by $f(\perp) = 0$, $f(a) = 1$ and $f(a') = 2$.



Preliminaries

Definition ($\mathcal{M}$-injective)

For a class $\mathcal{M}$ of monomorphisms in a category $\mathcal{C}$, an object $A \in \mathcal{C}$ is called *$\mathcal{M}$-injective* if for each $\mathcal{M}$-morphism $f : B \to C$ and morphism $g : B \to A$ there exists a morphism $h : C \to A$ such that $hf = g$.

$$\begin{tikzcd} B \arrow[r, "f \in \mathcal{M}"] \arrow[d, "g"'] & C \arrow[dl, "h"] \\ A & \end{tikzcd}$$

Preliminaries

Definition ($\mathcal{M}$-quasi injective)

An object $A \in \mathcal{C}$ is called *$\mathcal{M}$-quasi injective* if for each $\mathcal{M}$-morphism $m : B \to A$ and any morphism $f : B \to A$ there exists a morphism $\bar{f} : A \to A$ which extends f.

$$\begin{tikzcd} B \arrow[r, "m\in\mathcal{M}"] \arrow[d, "f"'] & A \arrow[dl, "\bar{f}"] \\ A & \end{tikzcd}$$

Preliminaries

Definition ($\mathcal{M}$-absolute retract)

An object $A \in \mathcal{C}$ is called $\mathcal{M}$-absolute retract if it is a retract of each of its $\mathcal{M}$-extensions; that is, for each $\mathcal{M}$-morphism $f : A \to C$ there exists a morphism $h : C \to A$ such that $hf = id_A$, in which case h is said to be a *retraction*.

$$\begin{tikzcd} A \arrow[r, "f \in \mathcal{M}"] \arrow[d, "id_A"'] & C \arrow[dl, "h"] \\ A & \end{tikzcd}$$

Theorem

$\mathcal{M}$*-injective* $\Longrightarrow$ $\mathcal{M}$*-absolute retract*

Preliminaries

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But the converse of above theorem, is not true in general. For the converse, we need some conditions.

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Theorem

In the category **Pos**-*S*,
$\mathcal{E}$-injective $\Longleftrightarrow$ $\mathcal{E}$-absolute retract

Some results about an $\mathcal{E}$-quasi injective S-poset

Theorem

In the category **Pos**-S,
$\mathcal{E}$-injective $\Rightarrow$ $\mathcal{E}$-quasi injective

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$\mathcal{E}$-injective $\not\Leftarrow$ $\mathcal{E}$-quasi injective

Theorem

In the category **Pos**-S,
Complete posets with identity actions $\Rightarrow$ $\mathcal{E}$-quasi injective

Some results about an $\mathcal{E}$-quasi injective S-poset

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Complete poset $\nRightarrow$ $\mathcal{E}$-quasi injective

Some results about an $\mathcal{E}$-quasi injective S-poset

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Complete poset $\not\Rightarrow$ $\mathcal{E}$-quasi injective

$$S\ (left\ zero)\quad \overset{s}{\bullet}\qquad \overset{e}{\bullet}\qquad \overset{t}{\bullet}$$

Some results about an $\mathcal{E}$-quasi injective S-poset

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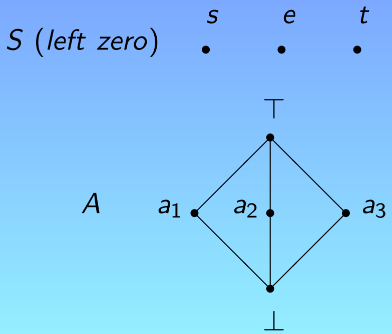
Complete poset $\not\Rightarrow$ $\mathcal{E}$-quasi injective



Some results about an $\mathcal{E}$-quasi injective S-poset

This poset A with the action

	s	t	e
$\bot$	$\bot$	$\bot$	$\bot$
a_1	a_1	a_1	a_1
a_2	a_1	a_3	a_2
a_3	a_3	a_3	a_3
$\top$	$\top$	$\top$	$\top$

is not $\mathcal{E}$-quasi injective.

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$\mathcal{E}$-quasi injective $\not\Rightarrow$ Complete poset

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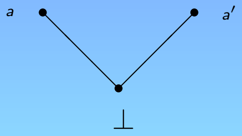

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The action on an $\mathcal{E}$-quasi injective S-poset need not be identity.

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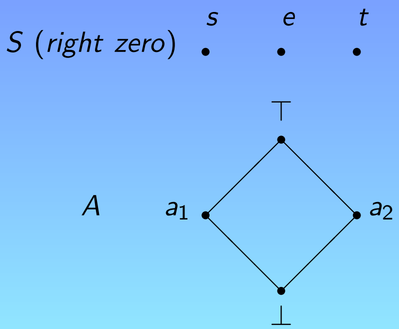

Some results about an $\mathcal{E}$-quasi injective S-poset

Then
$A^{(S)} = \{f : S \to A : f \ \textit{is a monotone map}\}$ with
• pointwise order, and
• the action given by $(fs)(t) = f(st)$, for $s, t \in S$ and $f \in A^{(S)}$
is $\mathcal{E}$-quasi injective and has non zero elements.

Some results about an $\mathcal{E}$-quasi injective S-poset

Proposition

There exists no po-monoid S over which all S-posets are $\mathcal{E}$-quasi injective.

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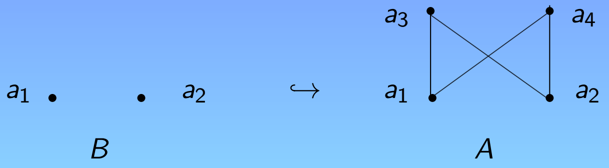

$f : B \to A$, $f(a_1) = a_3$, $f(a_2) = a_4$.
There does not exist an S-poset map $\bar{f} : A \to A$ which extends f.

Behaviour of $\mathcal{E}$-quasi injective S-posets with direct products

Theorem

Let $\{A_i : i \in I\}$ be a family of S-posets. If the product $\prod_{i\in I} A_i$ is $\mathcal{E}$-quasi injective in the category **Pos**-S and has a zero element $\theta = (\theta_i)_{i\in I}$ then each A_i is $\mathcal{E}$-quasi injective in **Pos**-S.

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But the converse is not true in general.

Remark

The product of $\mathcal{E}$-quasi injective S-posets, is not necessarily $\mathcal{E}$-quasi injective.

Relation between $\mathcal{E}$-quasi injectivity and $\mathcal{E}$-injectivity

Theorem

An $\mathcal{E}$-quasi injective S-poset A is $\mathcal{E}$-injective if and only if A has a zero element and $A \times \bar{A}^{(S)}$ is $\mathcal{E}$-quasi injective ($\bar{A}$ is the Dedekind-MacNeille completion of A).

A, $\mathcal{E}$-quasi injective $\not\Rightarrow$ $A \oplus \{\theta\}$, $\mathcal{E}$-quasi injective

Definition

An S-poset $A \oplus \{\theta\}$, which is obtained by adjoining a zero top element θ to an S-poset A, is called the θ-extension of A.

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$$S \ (left\ zero) \qquad \overset{s}{\bullet} \qquad \overset{e}{\bullet} \qquad \overset{t}{\bullet}$$

$$A \qquad \overset{a_1}{\bullet} \qquad \overset{a_2}{\bullet} \qquad \overset{a_3}{\bullet} \qquad \overset{a_4}{\bullet}$$

A, $\mathcal{E}$-quasi injective $\not\Rightarrow A \oplus \{\theta\}$, $\mathcal{E}$-quasi injective

The poset A with the following action is an $\mathcal{E}$-quasi injective S-poset. But, $A \oplus \{\theta\}$ is not $\mathcal{E}$-quasi injective.

	s	t	e
a_1	a_1	a_1	a_1
a_2	a_1	a_3	a_2
a_3	a_3	a_3	a_3
a_4	a_3	a_1	a_4

K_θ-quasi injectivity

Definition

An S-poset $A \oplus \{\theta\}$ is called *K_θ-quasi injective* if for every sub S-poset B of $A \oplus \{\theta\}$, any S-poset map $f : B \to A \oplus \{\theta\}$, with $\overleftarrow{f}(\theta) = \{b \in B : f(b) = \theta\} \neq \emptyset$, can be extended to $\bar{f} : A \oplus \{\theta\} \to A \oplus \{\theta\}$.

$$\begin{tikzcd} B \arrow[r] \arrow[d, "{f,\ with\ \overleftarrow{f}(\theta)\neq\emptyset}"'] & A \oplus \{\theta\} \arrow[dl, "\bar{f}"] \\ A \oplus \{\theta\} & \end{tikzcd}$$

S-filter

Definition

Let A be an S-act. A subset B of A is called *consistent* if for each $a \in A$ and $s \in S$, $as \in B$ implies $a \in B$. We call a consistent subact an *S-filter*.

S-filter

Definition

Let A be an S-act. A subset B of A is called *consistent* if for each $a \in A$ and $s \in S$, $as \in B$ implies $a \in B$. We call a consistent subact an *S-filter*.

Example

Whenever $S = G$ is a group, every subact B of a G-act A is a G-filter.

K_θ-quasi injectivity

Theorem

Let A be an $\mathcal{E}$-quasi injective S-poset. Also, assume that $f : B \to A \oplus \{\theta\}$ is an S-poset map, where B is a sub S-poset of $A \oplus \{\theta\}$ and $\overleftarrow{f}(\theta) \neq \emptyset$. Then there exists an S-filter $\tilde{A}$ of $A \oplus \{\theta\}$ which is

- upward closed in $A \oplus \{\theta\}$,
- $\overleftarrow{f}(\theta) \subseteq \tilde{A}$, and
- $\tilde{A} \cap \{b \in B : f(b) \neq \theta\} = \emptyset$

if and only if there exists an S-poset map $\bar{f} : A \oplus \{\theta\} \to A \oplus \{\theta\}$ which extends f.

K_θ-quasi injectivity

Corollary

Let A be an $\mathcal{E}$-quasi injective S-poset. If for each S-poset map $f : B \to A \oplus \{\theta\}$, where B is a sub S-poset of $A \oplus \{\theta\}$ and $\overleftarrow{f}(\theta) \neq \emptyset$, we have

$$\tilde{A} = \{a \in A \oplus \{\theta\} : \exists s \in S, as \in \uparrow \overleftarrow{f}(\theta)\}$$

is an S-filter of $A \oplus \{\theta\}$, then $A \oplus \{\theta\}$ is K_θ-quasi injective.

K_θ-quasi injectivity

Corollary

Suppose S is a po-monoid with any one of the following properties:
(1) $\forall s \in S$, $\exists t \in S$, $st \leq e$.
(2) $\forall s \in S$, $s^2 \leq e$.
(3) S is a po-group.
(4) $\top_S = e$ ($\top_S$ is the top element of S).
(5) S is a right zero semigroup with an adjoined identity.
If A is an $\mathcal{E}$-quasi injective S-poset then $A \oplus \{\theta\}$ is K_θ-quasi injective in **Pos**-S.

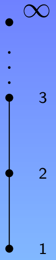
Decreasing action

Definition

Let A be an S-poset. Then, the action on A is called *decreasing* (*contractive*, *non expansive*) if for every $a \in A$ and $s \in S$, $as \leq a$.

Example for Decreasing action

$$S = \mathbb{N}^{\infty} = \mathbb{N} \cup \{\infty\}$$



• With the binary operation $m.n = min\{m, n\}$, S is a po-monoid and ∞ is the identity element of S.
• S has the properties (1), (2), and (4) in previous corollary.
• Also, every $\mathbb{N}^{\infty}$-poset has the property that $an \leq a\infty = a$, for all $n \in \mathbb{N}^{\infty}$. Therefore, the action on every $\mathbb{N}^{\infty}$-poset is decreasing.

K_θ-quasi injectivity

Corollary

Let $\mathcal{D}$ be a full subcategory of the category **Pos**-S, where the action on every object $A \in \mathcal{D}$ is decreasing. If A is $\mathcal{E}$-quasi injective in $\mathcal{D}$ then $A \oplus \{\theta\}$ is K_θ-quasi injective in $\mathcal{D}$.

Reversible automata

Revesible automata are important kinds of automata and have studied under various names. Glushkov in [3] called them *invertible* and Thierrin in [5] called them *locally transitive* (see also [1]). Since each S-act is a kind of automaton and because of the importance of reversible automata in automata theory, we introduce the category of reversible partially ordered acts and study the notion of $\mathcal{E}$-quasi injectivity in this category.

Reversible S-poset

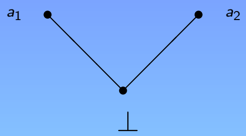
Definition

An S-poset A is called *reversible* if for every $a \in A$ and $s \in S$, there exists $t \in S$ such that $ast = a$. So, we have the category **R**-**Pos**-S of all reversible S-posets with S-poset maps between them.

Example of Reversible S-poset

$S(right\ \ zero)$ $\quad s \quad e \quad t$

A



	s	t	e
a_1	a_2	a_1	a_1
a_2	a_2	a_1	a_2
$\perp$	$\perp$	$\perp$	$\perp$

Strong Reversible S-poset

Definition

An S-poset A is called *strong reversible* if for every $s \in S$ there exists $t \in S$ such that $ast = ats = a$ for all $a \in A$. So, we have the category **SR-Pos**-S of all strong reversible S-posets and S-poset maps between them.

Example of Strong Reversible S-poset

S: $\overset{s}{\bullet}$ $\overset{e}{\bullet}$ $\overset{t}{\bullet}$ $\overset{v}{\bullet}$

	s	t	v	e
s	t	v	s	s
t	v	s	t	t
v	s	t	v	v
e	s	t	v	e

Example of Strong Reversible S-poset

A

	s	t	v	e
a	b	c	a	a
b	c	a	b	b
c	a	b	c	c

Square Reversible S-poset

Definition

An S-poset A is *square reversible* if for every $a \in A$ and $s \in S$, we have $as^2 = a$. So, we have the category **SQ-Pos**-S of all square reversible S-posets and S-poset maps between them.

Example of Suare Reversible S-poset

S $\quad \overset{s}{\bullet} \quad \overset{e}{\bullet} \quad \overset{t}{\bullet}$ $\qquad$ A $\quad \overset{a_1}{\bullet} \quad \overset{a_2}{\bullet}$

	s	t	e
s	t	s	s
t	s	t	t
e	s	t	e

	s	t	e
a_1	a_2	a_1	a_1
a_2	a_1	a_2	a_2

Relation between square, strong and reversible

Square reversible $\Rightarrow$ Strong reversible $\Rightarrow$ Reversible

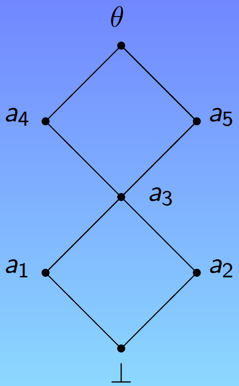
- But the converses are not true in general.
- S po-group $\Rightarrow$ every S-poset is strong reversible.
- S as an S-poset is strong reversible $\Longleftrightarrow$ S is a group.

K_θ-quasi injectivity

Corollary

Let A be an $\mathcal{E}$-quasi injective (strong, square) reversible S-poset. Then $A \oplus \{\theta\}$ is K_θ-quasi injective in **R-Pos**-S (**SR-Pos**-S, **SQ-Pos**-S).

A, K_θ-quasi injective $\not\Rightarrow A \setminus \{\theta\}$, $\mathcal{E}$-quasi injective



Relation between $\mathcal{E}$-quasi injectivity and completeness

Theorem

Let A be a strong (square) reversible S-poset. Then the following are equivalent in the category **SR-Pos**-S (**SQ-Pos**-S):
(i) A is $\mathcal{E}$-injective.
(ii) A is $\mathcal{E}$-absolute retract.
(ii) A is complete.

Relation between $\mathcal{E}$-quasi injectivity and completeness

Theorem

Let A be a strong (square) reversible S-poset. Then the following are equivalent in the category **SR**-**Pos**-S (**SQ**-**Pos**-S):
(i) A is $\mathcal{E}$-injective.
(ii) A is $\mathcal{E}$-absolute retract.
(ii) A is complete.

Corollary

(i) In **SR**-**Pos**-S, an object A is $\mathcal{E}$-quasi injective if it is complete.
(ii) In **SQ**-**Pos**-S, an object A is $\mathcal{E}$-quasi injective if it is complete.

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Thank You For Your Attention

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