Logics for extended distributive contact lattices

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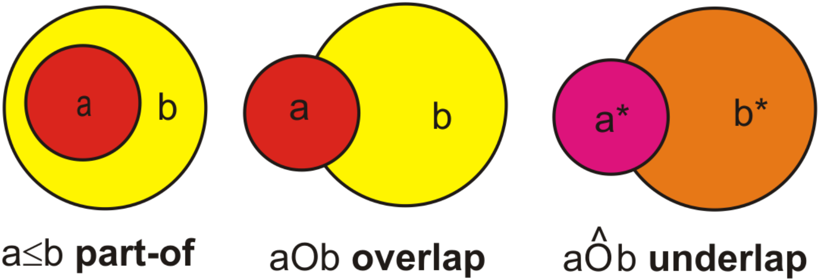
In the classical Euclidean geometry the notion of point is taken as one of the basic primitive notions. In contrast the region-based theory of space (RBTS) has as primitives the more realistic notion of region as an abstraction of physical body, together with some basic relations and operations on regions. Some of these relations are mereological - part-of ($x \leq y$), overlap (xOy), its dual underlap ($x\widehat{O}y$).



Basic mereological relations

Other relations are topological - contact (xCy), nontangential part-of ($x \ll y$), dual contact ($x\widehat{C}y$) and some others definable by means of the contact and part-of relations. This is one of the reasons that the extension of mereology with these new relations is commonly called mereotopology. The motivation for taking region as a primary notion is that points, lines and planes do not have separate existence in the reality. RBTS has simpler way of representing of qualitative spatial information.

Contact algebra

Contact algebra is one of the main tools in RBTS.
Contact algebra (Dimov and Vakarelov, 2006) is a Boolean algebra $\underline{B} = (B, \leq, 0, 1, \cdot, +, *, C)$ with an additional binary relation C called *contact*, and satisfying the following axioms:

(C1) If aCb, then $a \neq 0$ and $b \neq 0$,
(C2) If aCb and $a \leq a'$ and $b \leq b'$, then $a'Cb'$,
(C3) If $aC(b + c)$, then aCb or aCc,
(C4) If aCb, then bCa,
(C5) If $a \cdot b \neq 0$, then aCb.

The elements of contact algebra are called regions. Boolean operations are considered as operations for constructing new regions from given ones. The unit element 1 symbolizes the region containing as its parts all regions, and the zero region 0 symbolizes the non-existing region. The contact relation is used also to define non-tangential inclusion, dual contact in the following way: $a \ll b \leftrightarrow a\overline{C}b^*$, $a\widehat{C}b \leftrightarrow a^*Cb^*$.

Relational contact algebra

Let (W, R) be a relational system where W is a nonempty set and R is a reflexive and symmetric relation in W and let B be a set of subsets of W closed under union, intersection and compliment, containing $\emptyset$ and W. We define $0 = \emptyset$, $1 = W$. For arbitrary a, b in B we define:

$a \leq b$ iff $a \subseteq b$

$a \cdot b = a \cap b$

$a + b = a \cup b$

$a^* = W - a$

Define a contact relation between a and b as follows

(Def C_R) aC_Rb iff $\exists x \in a$ and $\exists y \in b$ such that xRy.

Topological contact algebra

Let X be a topological space and $a \subseteq X$. We say that a is a *regular closed set* if $a = Cl(Int(a))$ and a is a *regular open* set if $a = Int(Cl(a))$. It is a well known fact that the set $RC(X)$ of all regular closed subsets of X is a Boolean algebra with respect to the relations, operations and constants defined as follows:
$a \leq b$ iff $a \subseteq b$, $0 = \varnothing$, $1 = X$, $a + b = a \cup b$,
$a \cdot b = Cl(Int(a \cap b))$, $a^* = Cl(-a)$ where $-a = X \setminus a$. If we define a contact C by aCb iff $a \cap b \neq \varnothing$ then we obtain the standard topological model of contact algebra.
Another topological model of contact algebra is by the set $RO(X)$ of regular open subsets of X. This contact algebra is defined dually to the contact algebra of the regular closed subsets of X.

Extended distributive contact lattice

There is a problem in the motivation of the operation of Boolean complementation. A question arises: if *a* represents some physical body, what kind of body represents a^* - it depends on the universe in which we consider *a*. To avoid this problem, we drop the operation $*$. The topological relations of dual contact and nontangential inclusion cannot be defined without $*$ and because of this we take them as primary in the language. So we consider the language $\mathcal{L}(0, 1; +, \cdot; \leq, C, \widehat{C}, \ll)$ which is an extension of the language of distributive lattice with the predicate symbols for the relations of contact, dual contact and nontangential inclusion. We obtain an axiomatization of the theory consisting of the formulas in the language $\mathcal{L}$ true in all contact algebras. The structures in $\mathcal{L}$, satisfying the axioms in question, are called extended distributive contact lattices (EDC-lattices).

Principle of duality

For the language we can introduce the following principle of duality: dual pairs $(0, 1), (\cdot, +), (\leq, \geq), (C, \widehat{C}), (\ll, \gg)$. For each formula A of the language we can define in an obvious way its dual $\widehat{A}$.

Extended distributive contact lattice

Let $\underline{D} = (D, \leq, 0, 1, \cdot, +, C, \widehat{C}, \ll)$ be a bounded distributive lattice with three additional relations $C, \widehat{C}, \ll$, called respectively **contact**, **dual contact** and **nontangential part-of.** The obtained system is called **extended distributive contact lattice** (EDC-lattice, for short) if it satisfies the following axioms:

Extended distributive contact lattice

Axioms for C alone: The axioms (C1)-(C5) mentioned above.

Axioms for $\widehat{C}$ alone:
($\widehat{C}$1) If $a\widehat{C}b$, then $a, b \neq 1$,
($\widehat{C}$2) If $a\widehat{C}b$ and $a' \leq a$ and $b' \leq b$, then $a'\widehat{C}b'$,
($\widehat{C}$3) If $a\widehat{C}(b \cdot c)$, then $a\widehat{C}b$ or $a\widehat{C}c$,
($\widehat{C}$4) If $a\widehat{C}b$, then $b\widehat{C}a$,
($\widehat{C}$5) If $a + b \neq 1$, then $a\widehat{C}b$.

Extended distributive contact lattice

Axioms for $\ll$ alone:

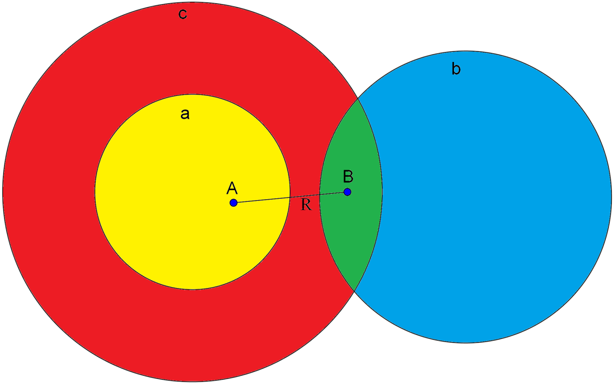
($\ll$ 1) $0 \ll 0$,
($\ll$ 2) $1 \ll 1$,
($\ll$ 3) If $a \ll b$, then $a \leq b$,
($\ll$ 4) If $a' \leq a \ll b \leq b'$, then $a' \ll b'$,
($\ll$ 5) If $a \ll c$ and $b \ll c$, then $(a + b) \ll c$,
($\ll$ 6) If $c \ll a$ and $c \ll b$, then $c \ll (a \cdot b)$,
($\ll$ 7) If $a \ll b$ and $(b \cdot c) \ll d$ and $c \ll (a + d)$, then $c \ll d$.

Extended distributive contact lattice

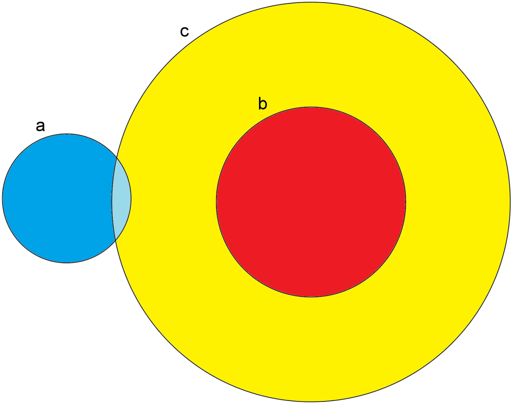
Mixed axioms:

$(MC1)$ If aCb and $a \ll c$, then $aC(b \cdot c)$,
$(MC2)$ If $a\overline{C}(b \cdot c)$ and aCb and $(a \cdot d)\overline{C}b$, then $d\widehat{C}c$,
$(M\widehat{C}1)$ If $a\widehat{C}b$ and $c \ll a$, then $a\widehat{C}(b + c)$,
$(M\widehat{C}2)$ If $a\overline{\widehat{C}}(b + c)$ and $a\widehat{C}b$ and $(a + d)\overline{\widehat{C}}b$, then dCc,
$(M \ll 1)$ If $a\overline{\widehat{C}}b$ and $(a \cdot c) \ll b$, then $c \ll b$,
$(M \ll 2)$ If $a\overline{C}b$ and $b \ll (a + c)$, then $b \ll c$.

(*MC*1) If aCb and $a \ll c$, then $aC(b \cdot c)$



$(M \ll 2)$ If $a\overline{C}b$ and $b \ll (a + c)$, then $b \ll c$



For each axiom Ax from the list of axioms of EDCL its dual $\widehat{Ax}$ is also an axiom.

The leading intuition to accept exactly these axioms is the following: (1) the axioms to be first-order sentences true in contact algebras, (2) the principle of duality to be true and (3) the axioms to be sufficient to prove the embedding theorem of EDC-latices in contact algebras.

The following statements are well known in the representation theory of distributive lattices.

Lemma

Let F_0 be a filter, I_0 be an ideal and $F_0 \cap I_0 = \varnothing$. Then:

1. **Filter-extension Lemma.** *There exists a prime filter F such that $F_0 \subseteq F$ and $F \cap I_0 = \varnothing$.*
2. **Ideal-extension Lemma.** *There exists a prime ideal I such that $I_0 \subseteq I$ and $F_0 \cap I = \varnothing$.*

There are also stronger filter-extension lemma and ideal-extension lemma. We do not know if these two statements for distributive lattices are new, but we use them in the representation theorem of EDC-lattices.

Lemma

(Ivanova and Vakarelov, 2016) Let F_0 be a filter, I_0 be an ideal and $F_0 \cap I_0 = \varnothing$. Then:

1. **Strong filter-extension Lemma.** *There exists a prime filter F such that $F_0 \subseteq F$, $F \cap I_0 = \varnothing$ and $(\forall x \notin F)(\exists y \in F)(x \cdot y \in I_0)$.*
2. **Strong ideal-extension Lemma.** *There exists a prime ideal I such that $I_0 \subseteq I$, $F_0 \cap I = \varnothing$ and $(\forall x \notin I)(\exists y \in I)(x + y \in F_0)$.*

Canonical relational structure

Let $\underline{D} = (D, C, \widehat{C}, \ll)$ be an EDC-lattice and let $PF(D)$ denote the set of all prime filters of $\underline{D}$. We construct a canonical relational structure (W, R) related to $\underline{D}$ putting $W = PF(D)$ and defining the canonical relation R for $\Gamma, \Delta \in PF(D)$ as follows:

$\Gamma R \Delta \leftrightarrow_{def} (\forall a, b \in D)((a \in \Gamma, b \in \Delta \rightarrow aCb)\&(a \notin \Gamma, b \notin \Delta \rightarrow a\widehat{C}b)\&(a \in \Gamma, b \notin \Delta \rightarrow a \not\ll b)\&(a \notin \Gamma, b \in \Delta \rightarrow b \not\ll a))$

Let $h(a) = \{\Gamma \in PF(D) : a \in \Gamma\}$ be the well known Stone embedding mapping. It turns out that h is an embedding from $\underline{D}$ into the EDC-lattice over (W, R).

Corollary

Every EDC-lattice can be isomorphically embedded into a contact algebra.

We compare EDC-lattices with other two mereotopologies: the *relational mereotopology* and *RCC-8*.
Relational mereotopology is based on *mereotopological structures* introduced in

Y. Nenov, D. Vakarelov. Modal logics for mereotopological relations, *Advances in Modal Logic*, volume 7, College Publications 2008, 249-272

These are relational structures in the form $(W, \leq, O, \widehat{O}, \ll, C, \widehat{C})$ axiomatizing the basic mereological relations part-of $\leq$, overlap O and dual overlap (underlap) $\widehat{O}$, and the basic mereotopological relations non-tangential part-of $\ll$, contact C and dual contact $\widehat{C}$. These relations satisfy the following list of universal first-order axioms:

Relations with other mereotopologies

$$
\begin{array}{ll}
(\leq 0) & a \leq b \text{ and } b \leq a \rightarrow a = b, \\
(\leq 1) & a \leq a, \\
(\leq 2) & a \leq b \text{ and } b \leq c \rightarrow a \leq c, \\
(O1) & aOb \rightarrow bOa, \\
(\widehat{O}1) & a\widehat{O}b \rightarrow b\widehat{O}a, \\
(O2) & aOb \rightarrow aOa, \\
(\widehat{O}2) & a\widehat{O}b \rightarrow a\widehat{O}a, \\
(\overline{O} \leq) & a\overline{O}a \rightarrow a \leq b, \\
(\overline{\widehat{O}} \leq) & b\overline{\widehat{O}}b \rightarrow a \leq b, \\
(O \leq) & aOb \text{ and } b \leq c \rightarrow aOc, \\
(\widehat{O} \leq) & c \leq a \text{ and } a\widehat{O}b \rightarrow c\widehat{O}b, \\
(O\widehat{O}) & aOa \text{ or } a\widehat{O}a, \\
(\leq O\widehat{O}) & c\overline{O}a \text{ and } c\overline{\widehat{O}}b \rightarrow a \leq b,
\end{array}
$$

Relations with other mereotopologies

$$
\begin{array}{ll}
(C) & aCb \rightarrow bCa, \\
(\widehat{C}) & a\widehat{C}b \rightarrow b\widehat{C}a, \\
(CO1) & aOb \rightarrow aCb, \\
(\widehat{C}\widehat{O}1) & a\widehat{O}b \rightarrow a\widehat{C}b, \\
(CO2) & aCb \rightarrow aOa, \\
(\widehat{C}\widehat{O}2) & a\widehat{C}b \rightarrow a\widehat{O}a, \\
(C \leq) & aCb \text{ and } b \leq c \rightarrow aCc, \\
(\widehat{C} \leq) & a\widehat{C}b \text{ and } c \leq b \rightarrow a\widehat{C}c, \\
(\ll\leq 1) & a \ll b \rightarrow a \leq b, \\
(\ll\leq 2) & a \leq b \text{ and } b \ll c \rightarrow a \ll c, \\
(\ll\leq 3) & a \ll b \text{ and } b \leq c \rightarrow a \ll c, \\
(\ll O) & a\overline{O}a \rightarrow a \ll b, \\
(\ll \widehat{O}) & b\overline{\widehat{O}}b \rightarrow a \ll b
\end{array}
$$

$$
\begin{array}{ll}
(\ll CO) & aCb \text{ and } b \ll c \rightarrow aOc, \\
(\ll \widehat{C}\widehat{O}) & c \ll a \text{ and } a\widehat{C}b \rightarrow c\widehat{O}b, \\
(\ll C\widehat{O}) & c\overline{C}a \text{ and } c\overline{\widehat{O}}b \rightarrow a \ll b, \\
(\ll \widehat{C}O) & c\overline{O}a \text{ and } c\overline{\widehat{C}}b \rightarrow a \ll b.
\end{array}
$$

Relations with other mereotopologies

The following theorem relates EDC-lattices to mereotopological structures.

Theorem

Every EDC-lattice is a mereotopological structure under the standard definitions of the basic mereological relations.

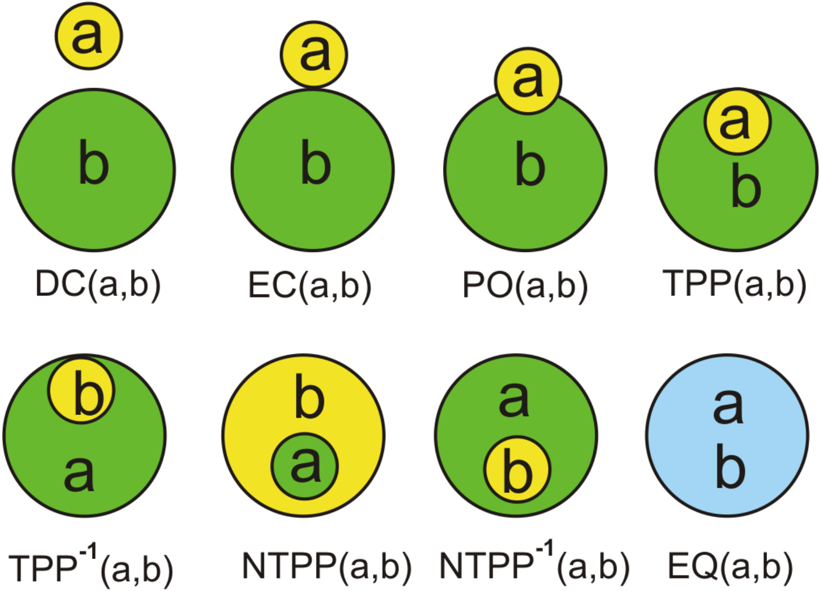
$aOb \leftrightarrow a.b \neq 0$
$a\hat{O}b \leftrightarrow a + b \neq 1$

One of the most popular systems of topological relations in Qualitative Spatial Representation and Reasoning is RCC-8. It consists of 8 relations between non-empty regular closed subsets of arbitrary topological space.

Definition

The system **RCC-8.**

- disconnected – DC(a, b): $a\overline{C}b$,
- external contact – EC(a, b): aCb and $a\overline{O}b$,
- partial overlap – PO(a, b): aOb and $a \not\leq b$ and $b \not\leq a$,
- tangential proper part – TPP(a, b): $a \leq b$ and $a \not\ll b$ and $b \not\leq a$,
- tangential proper part^{-1} – TPP$^{-1}(a, b)$: $b \leq a$ and $b \not\ll a$ and $a \not\leq b$,
- nontangential proper part NTPP(a, b): $a \ll b$ and $a \neq b$,
- nontangential proper part^{-1} – NTPP$^{-1}(a, b)$: $b \ll a$ and $a \neq b$,
- equal – EQ(a, b): $a = b$.



RCC 8 topological relations

Relations with other mereotopologies

The *RCC*-8 relations are definable in the language of *EDC*-lattices.

Additional axioms

We formulate several additional axioms for EDC-lattices which are adaptations for the language of EDC-lattices of some known axioms considered in the context of contact algebras. First we formulate the so called extensionality axioms for the definable predicates of overlap - $aOb \leftrightarrow_{def} a \cdot b \neq 0$ and underlap - $a\widehat{O}b \leftrightarrow_{def} a + b \neq 1$.

(Ext O) $a \not\leq b \rightarrow (\exists c)(a \cdot c \neq 0$ and $b \cdot c = 0)$ - *extensionality of overlap*,
(Ext $\widehat{O}$) $a \not\leq b \rightarrow (\exists c)(a + c = 1$ and $b + c \neq 1)$ - *extensionality of underlap*.

Additional axioms

(Ext C) $a \neq 1 \to (\exists b \neq 0)(a\overline{C}b)$ - *C-extensionality*,

(Ext $\widehat{C}$) $a \neq 0 \to (\exists b \neq 1)(a\overline{\widehat{C}}b)$ - *$\widehat{C}$-extensionality*.

(Con C) $a \neq 0, b \neq 0$ and $a + b = 1 \to aCb$ - *C-connectedness axiom* ,

(Con $\widehat{C}$) $a \neq 1, b \neq 1$ and $a \cdot b = 0 \to a\widehat{C}b$ - *$\widehat{C}$-connectedness axiom* .

Additional axioms

(Nor 1) $a\overline{C}b \rightarrow (\exists c, d)(c + d = 1, a\overline{C}c$ and $b\overline{C}d)$,

(Nor 2) $a\overline{\widehat{C}}b \rightarrow (\exists c, d)(c \cdot b = 0, a\overline{\widehat{C}}c$ and $b\overline{\widehat{C}}d)$,

(Nor 3) $a \ll b \rightarrow (\exists c)(a \ll c \ll b)$.

and the so called rich axioms:

(U-rich $\ll$) $a \ll b \rightarrow (\exists c)(b + c = 1$ and $a\overline{C}c)$,

(U-rich $\widehat{C}$) $a\overline{\widehat{C}}b \rightarrow (\exists c, d)(a + c = 1, b + d = 1$ and $c\overline{C}d)$.

(O-rich $\ll$) $a \ll b \rightarrow (\exists c)(a \cdot c = 0$ and $c\overline{\widehat{C}}b)$,

(O-rich C) $a\overline{C}b \rightarrow (\exists c, d)(a \cdot c = 0, b \cdot d = 0$ and $c\overline{\widehat{C}}d)$.

Theorem

Topological representation theorem for EDC-lattices.

(Ivanova and Vakarelov, 2016) Let $\underline{D} = (D, C, \widehat{C}, \ll)$ *be an EDC-lattice. Then there exists a compact semiregular* T_0 *topological space* X *and an embedding of* $\underline{D}$ *into the contact algebra* $RC(X)$ *of regular closed subsets of* X*.*

The following lemma relates topological properties to the properties of the relations C, $\widehat{C}$ and $\ll$ and shows the importance of the additional axioms for EDC-lattices.

Lemma

(i) If X is semiregular, then X is weakly regular iff $RC(X)$ satisfies any of the axioms (Ext C), (Ext $\widehat{C}$).
(ii) X is κ-normal iff $RC(X)$ satisfies any of the axioms (Nor 1), (Nor 2) and (Nor 3).
(iii) X is connected iff $RC(X)$ satisfies any of the axioms (Con C), (Con $\widehat{C}$).
(iv) If X is compact and Hausdorff, then $RC(X)$ satisfies (Ext C), (Ext $\widehat{C}$) and (Nor 1), (Nor 2) and (Nor 3) .

Definition

U-rich and O-rich EDC-lattices. Let $\underline{D} = (D, C, \widehat{C}, \ll)$ be an EDC-lattice. Then:
(i) $\underline{D}$ is called U-rich EDC-lattice if it satisfies the axioms (Ext $\widehat{O}$), (U-rich $\ll$) and (U-rich $\widehat{C}$).
(ii) $\underline{D}$ is called O-rich EDC-lattice if it satisfies the axioms (Ext O), (O-rich $\ll$) and (O-rich $\widehat{C}$).

Topological representation theory

Theorem

Topological representation theorem for U-rich EDC-lattices
(Ivanova and Vakarelov, 2016)
Let $\underline{D} = (D, C, \widehat{C}, \ll)$ be an U-rich EDC-lattice. Then there exists a compact semiregular T_0-space X and an embedding of $\underline{D}$ into the Boolean contact algebra $RC(X)$ of the regular closed sets of X. Moreover:
(i) $\underline{D}$ satisfies (Ext C) iff $RC(X)$ satisfies (Ext C); in this case X is weakly regular.
(ii) $\underline{D}$ satisfies (Con C) iff $RC(X)$ satisfies (Con C); in this case X is connected.
(iii) $\underline{D}$ satisfies (Nor 1) iff $RC(X)$ satisfies (Nor 1); in this case X is κ-normal.

There is also a topological representation theorem of U-rich EDC-lattices, satisfying (Ext C), in T_1-spaces.

Theorem

Topological representation theorem for C-extensional *U*-rich EDC-lattices *(Ivanova and Vakarelov, 2016) Let* $\underline{D} = (D, C, \widehat{C}, \ll)$ *be a C-extensional U-rich EDC-lattice. Then there exists a compact weakly regular* T_1*-space* X *and an embedding of* $\underline{D}$ *into the Boolean contact algebra* $RC(X)$ *of the regular closed sets of* X*. Moreover:*
(i) $\underline{D}$ *satisfies (Con C) iff* $RC(X)$ *satisfies (Con C); in this case* X *is connected.*
(ii) $\underline{D}$ *satisfies (Nor 1) iff* $RC(X)$ *satisfies (Nor 1); in this case* X *is* κ*-normal.*

Adding the axiom (Nor 1), we obtain representability in compact T_2-spaces.

Theorem

Topological representation theorem for U-rich EDC-lattices satisfying (Ext C) and (Nor 1). *(Ivanova and Vakarelov, 2016)* *Let $\underline{D} = (D, C, \widehat{C}, \ll)$ be an U-rich EDC-lattice satisfying (Ext C) and (Nor 1). Then there exists a compact T_2-space X and an embedding of $\underline{D}$ into the Boolean contact algebra $RC(X)$ of the regular closed sets of X. Moreover $\underline{D}$ satisfies (Con C) iff $RC(X)$ satisfies (Con C) and in this case X is connected.*

We consider a first-order language without quantifiers corresponding to EDCL. We give completeness theorems with respect to both algebraic and topological semantics for several logics for this language. It turns out that all these logics are decidable.

Logics for EDCL

We consider the quantifier-free first-order language with equality $\mathcal{L}$ which includes:
• constants: 0, 1;
• function symbols: $+$, $\cdot$;
• predicate symbols: $\leq$, C, $\widehat{C}$, $\ll$.

We consider the logic L with rule MP and the following axioms:
• the axioms of the classical propositional logic;
• the axiom schemes of distributive lattice;
• the axioms for C, $\widehat{C}$, $\ll$ and the mixed axioms of EDCL - considered as axiom schemes.

We consider the following additional rules and an axiom scheme:

(R Ext $\widehat{O}$) $\frac{\alpha\to(a+p\neq 1\vee b+p=1) \text{ for all variables } p}{\alpha\to(a\leq b)}$,

(R U-rich $\ll$) $\frac{\alpha\to(b+p\neq 1\vee aCp) \text{ for all variables } p}{\alpha\to(a\overline{\ll}b)}$,

(R U-rich $\widehat{C}$) $\frac{\alpha\to(a+p\neq 1\vee b+q\neq 1\vee pCq) \text{ for all variables } p,\ q}{\alpha\to a\widehat{C}b}$,

(R Ext C) $\frac{\alpha\to(p\neq 0\to aCp) \text{ for all variables } p}{\alpha\to(a=1)}$,

(R Nor1) $\frac{\alpha\to(p+q\neq 1\vee aCp\vee bCq) \text{ for all variables } p,\ q}{\alpha\to aCb}$,

where α is a formula, a, b are terms

(Con C) $p\neq 0\wedge q\neq 0\wedge p+q=1\to pCq$

Logics for EDCL

Let L' be for example the extension of L with the rule (R Ext $\widehat{O}$) and the axiom scheme (Con C). Then we denote L' by $L_{ConC,Ext\widehat{O}}$ and call the axioms (Con C) and (Ext $\widehat{O}$) additional axioms corresponding to L'. In a similar way we denote any extension of L with some of the considered additional rules and axiom scheme and in a similar way we define its corresponding additional axioms.

Theorem (Completeness theorem)

Let L' be some extension of L with 0 or more of the considered additional rules and axiom scheme. The following conditions are equivalent for any formula α:
(i) α is a theorem of L';
(ii) α is true in all EDCL, satisfying the additional axioms corresponding to L'.

We consider the following logics, corresponding to the considered EDC-lattices:

1) L;
2) $L_{Ext\widehat{O},U-rich\ll,U-rich\widehat{C}}$;
3) $L_{Ext\widehat{O},U-rich\ll,U-rich\widehat{C},ExtC}$;
4) $L_{Ext\widehat{O},U-rich\ll,U-rich\widehat{C},ConC}$;
5) $L_{Ext\widehat{O},U-rich\ll,U-rich\widehat{C},Nor1}$;
6) $L_{Ext\widehat{O},U-rich\ll,U-rich\widehat{C},ExtC,ConC}$;
7) $L_{Ext\widehat{O},U-rich\ll,U-rich\widehat{C},Nor1,ConC}$;
8) $L_{Ext\widehat{O},U-rich\ll,U-rich\widehat{C},ExtC,Nor1}$;
9) $L_{Ext\widehat{O},U-rich\ll,U-rich\widehat{C},ExtC,ConC,Nor1}$.

To every of these logics we juxtapose a class of topological spaces:
1) the class of all T_0, semiregular, compact topological spaces;
2) the class of all T_0, semiregular, compact topological spaces;
3) the class of all T_0, compact, weakly regular topological spaces;
4) the class of all T_0, semiregular, compact, connected topological spaces;
5) the class of all T_0, semiregular, compact, κ - normal topological spaces;

6) the class of all T_0, compact, weakly regular, connected topological spaces;
7) the class of all T_0, semiregular, compact, κ - normal, connected topological spaces;
8) the class of all T_0, compact, weakly regular, κ - normal topological spaces;
9) the class of all T_0, compact, weakly regular, connected, κ - normal topological spaces.

Theorem (Completeness theorem with respect to topological semantics)

Let L' be any of the considered logics. The following conditions are equivalent for any formula α:
(i) α is a theorem of L';
(ii) α is true in all contact algebras over a topological space from the class corresponding to L'.

Proposition

The following conditions are equivalent for any formula α:
(i) α is true in all EDCL;
(ii) α is true in all finite EDCL with number of the elements less or equal to $2^{2^n-1}+1$, where n is the number of the variables of α.

Corollary

L is decidable.

Some of the rules can be eliminated and using this fact we obtain:

Corollary

(i) The logics L, $L_{Ext\widehat{O},U-rich\ll,U-rich\widehat{C}}$, $L_{Ext\widehat{O},U-rich\ll,U-rich\widehat{C},ExtC}$, $L_{Ext\widehat{O},U-rich\ll,U-rich\widehat{C},Nor1}$, $L_{Ext\widehat{O},U-rich\ll,U-rich\widehat{C},ExtC,Nor1}$ *have the same theorems and are decidable;*
(ii) The logics $L_{ConC,U-rich\ll}$, $L_{Ext\widehat{O},U-rich\ll,U-rich\widehat{C},ConC}$, $L_{Ext\widehat{O},U-rich\ll,U-rich\widehat{C},ConC,Nor1}$, $L_{Ext\widehat{O},U-rich\ll,U-rich\widehat{C},ExtC,ConC}$, $L_{Ext\widehat{O},U-rich\ll,U-rich\widehat{C},ExtC,ConC,Nor1}$ *have the same theorems and are decidable.*

Thank you very much!