Reduced Rickart rings and skew nearlattices

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Outline

Motivation

- Any m-domain ring can be decomposed into disjoint semigroups – see [Subrahmanyam 1960].
- It can be proved that reduced Rickart rings and m-domain rings are the same thing.
- Any reduced Rickart ring R has a structure of skew nearlattice – see [Cīrulis 2015].

Question

- What happens to the skew nearlattice structure when we decompose the ring into semigroups?

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Answers

- We get a structure of strong semilattice of semigroups.
- The semigroups are actually skew nearlattices.
- When we try to "reconstruct" a reduced Rickart ring from its strong semilattice of semigroups, we get a reduced Baer semigroup.

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Rickart rings

Definition

A semigroup S is called Baer semigroup iff for every $a \in S$ there are idempotents $e, f \in S$ such that, for all $x \in S$,

- $ax = 0$ iff $ex = x$,
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Reduced Rickart rings

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A ring is called reduced iff it has no non-zero nilpotent elements.

Proposition

On a reduced Rickart ring, the idempotents e and f from the definition are unique and coincide.

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Focal operation

Definition

For every a in a reduced Rickart ring R, let a' be the unique idempotent such that

$$ax = 0 \iff a'x = x$$

for all x. The operation $'$ is called focal operation.

Examples of reduced Rickart rings

- $\mathbb{Z}$
- Any Boolean ring
- $\mathbb{Z}_{pq}$ for prime numbers $p \neq q$

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Abian order

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On a reduced ring, the following relation is a partial order:

$$a \leq b \text{ iff } ab = a^2$$

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It is called the Abian order.

- A reduced Rickart ring ordered by the Abian order is a semi-Boolean algebra. [Janowitz 1976]

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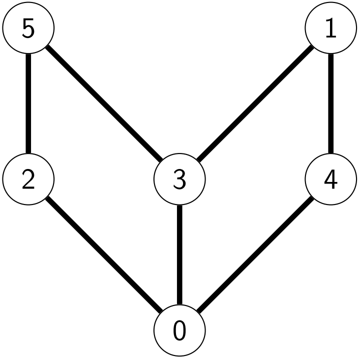
Example: The Abian order of $\mathbb{Z}_6$ and the focal operation

- Abian order of $\mathbb{Z}_6$
- Lattice of idempotents E
- The focal operation.



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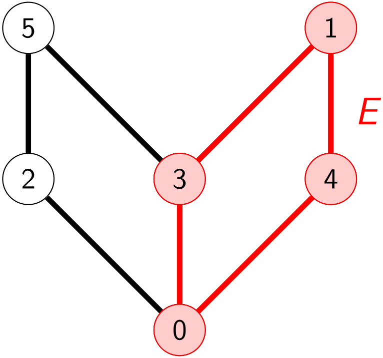

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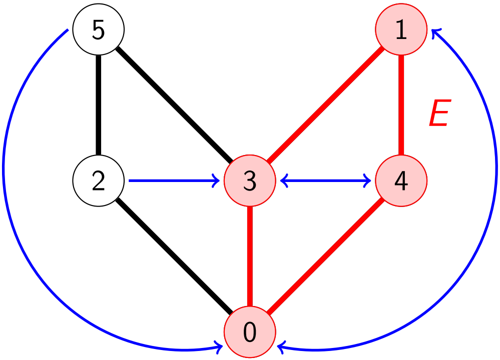

Nearlattices

Definition

A poset P is called nearlattice if

- P is a meet-semilattice,
- if $x, y \in P$ have an upper bound, then they have the join (i.e., P is finitely bounded complete).

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Natural order on a band

Definition

A semigroup is called band if all its elements are idempotent.

- Let $\langle S, \circ \rangle$ be a band.
- Let $x \leq_\circ y$ iff $xy = x = yx$.
- Then $\leq_\circ$ is a partial order, called the natural order of the band S.

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Right skew nearlattices ([Cīrulis 2015])

Definition

A (right) skew nearlattice is a partial algebra $\langle S, *, \vee \rangle$ such that

- There is a finitely bounded complete order $\leq$ on S
- $\vee$ is the respective partial join operation
- $*$ is an associative operation
- $x \vee y = y$ if and only if $x * y = x$.

- The operation $*$ is idempotent.
- The natural order on the band $\langle S, * \rangle$ is $\leq$.

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Example: Partial functions

- Let I, V be sets.
- Let $\mathcal{F}(\mathcal{I}, \mathcal{V})$ be the set of partial functions from I to V.
- For partial functions f, g, define

$$f \overleftarrow{\cap} g := g|_{\mathrm{dom}(f) \cap \mathrm{dom}(g)}.$$

- Then $\langle \mathcal{F}(\mathcal{I}, \mathcal{V}), \overleftarrow{\cap} \rangle$ is a band.
- The union of sets is their join with respect to the natural order of the band.
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Example in a Rickart ring ([Cīrulis 2015])

- Let R be a reduced Rickart ring.
- Define the skew meet:

$$a \overleftarrow{\wedge} b := a'' b$$

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- $\langle R, \overleftarrow{\wedge} \rangle$ is a band.
- Its natural order coincides with the Abian order.
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Right zero bands and right singular skew nearlattices

Definition

- A band $\langle S, \circ \rangle$ is called right zero band if

$$x \circ y = y$$

for all $x, y \in S$.

- A skew nearlattice $\langle S, *, \vee \rangle$ is called right singular if its band reduct $\langle S, * \rangle$ is right zero.

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Right singular skew nearlattices in a reduced Rickart ring

Proposition

- *Let R be a reduced Rickart ring.*
- *Let U be the set of non-zero divisors.*
- *Let e be an idempotent.*
- *Let $a \overleftarrow{\wedge} b := a''b$.*
- *Then $\langle eU, \overleftarrow{\wedge}, \vee \rangle$ is a right singular skew nearlattice.*

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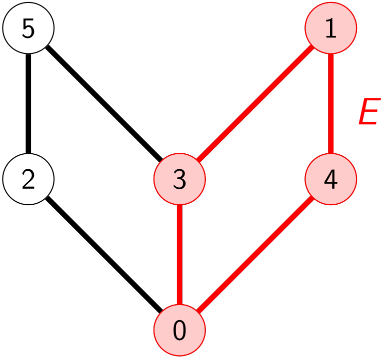
Example: The singular skew nearlattices $\langle eU, \overleftarrow{\wedge}, \vee \rangle$ in $\mathbb{Z}_6$

- Abian order of $\mathbb{Z}_6$
- Lattice of idempotents E
- The set of non-zero divisors U
- The other right singular skew nearlatttices eU



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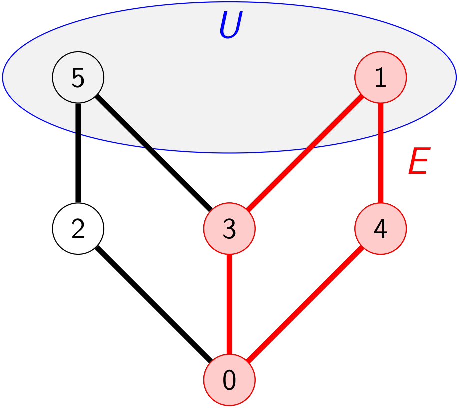

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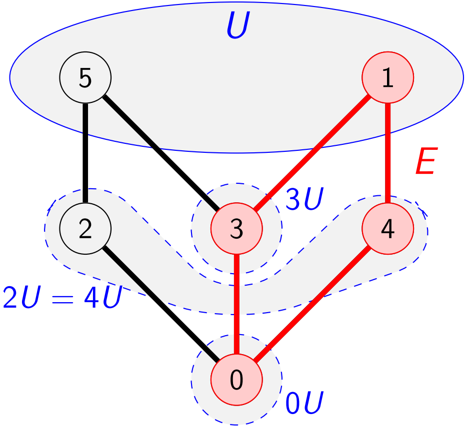

Homomorphisms of skew nearlattices

Definition

- Let $\langle A, *_A, \vee_A \rangle$ and $\langle B, *_B, \vee_B \rangle$ be skew nearlattices.
- A map $f : A \to B$ is called homomorphism of skew nearlattices if
  - $f : \langle A, *_A \rangle \longrightarrow \langle B, *_B \rangle$ is a semigroup homomorphism,
  - whenever $x, y \in A$ have the join, so do their images, and

$$f(x \vee y) = f(x) \vee f(y).$$

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Strong semilattice of semigroups

Definition (Clifford)

- Let T be a meet-semilattice,
- Let $\{\langle A_s, *_s \rangle \mid s \in T\}$ be a family of disjoint semigroups,
- Suppose that, for all $s, t \in T$ with $s \leq t$, there are semigroup homomorphisms $f_s^t : A_t \to A_s$ such that
- f_t^t are the identity maps
- if $r \leq s \leq t$, then $f_r^s f_s^t = f_r^t$.
- Let $A := \bigcup_{s \in T} A_s$.
- For $x \in A_s, y \in A_t$define $x \overleftarrow{\wedge} y := f_{s\wedge t}^s(x) *_{s\wedge t} f_{s\wedge t}^t(y)$.
- The algebra $\left\langle A, \overleftarrow{\wedge} \right\rangle$ is called a strong semilattice of the semigroups $\{A_s\}_{s\in T}$.

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Definition

A partial algebra $\langle S, *, \vee, \cdot \rangle$ will be called multiplicative skew nearlattice if

- $\langle S, *, \vee \rangle$ is a skew nearlattice,
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Multiplicative skew nearlattices in a reduced Rickart ring

Theorem

- *Let R be a reduced Rickart ring.*
- *Let U be the set of non-zero divisors.*
- *Let e be an idempotent.*
- *Let $a \overleftarrow{\wedge} b := a''b$.*
- *Then $\langle eU, \overleftarrow{\wedge}, \vee, \cdot \rangle$ is a multiplicative right singular skew nearlattice.*
- *Moreover, $\left\langle R, \overleftarrow{\wedge}, \cdot \right\rangle$ is a strong semilattice of the multiplicative skew nearlattices $\left\langle eU, \vee, \overleftarrow{\wedge}, \cdot \right\rangle$.*

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Example: The reduced Rickart ring $\mathbb{Z}_6$

- Abian order of $\mathbb{Z}_6$
- Lattice of idempotents E
- The set of non-zero divisors U
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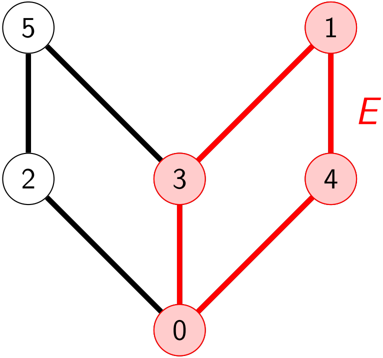
- The homomorphisms of skew nearlattices



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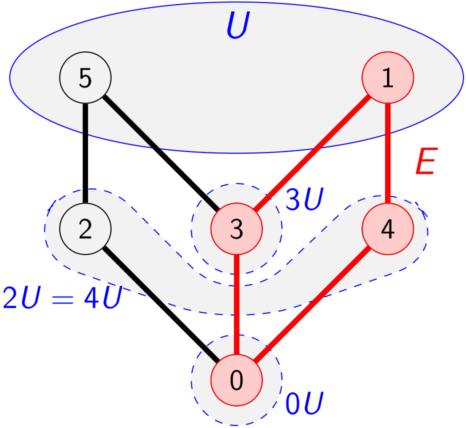

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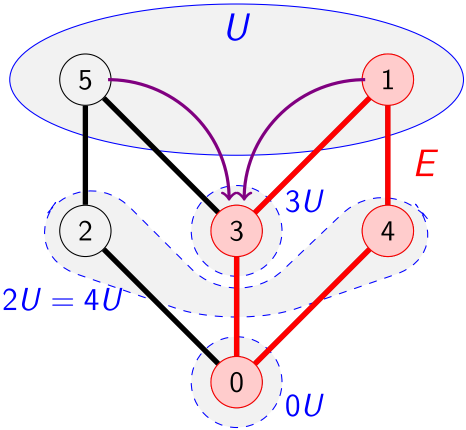

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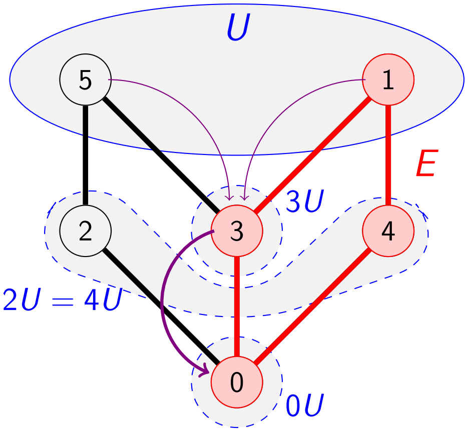

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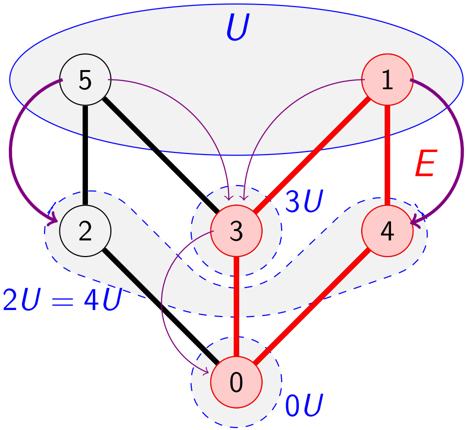

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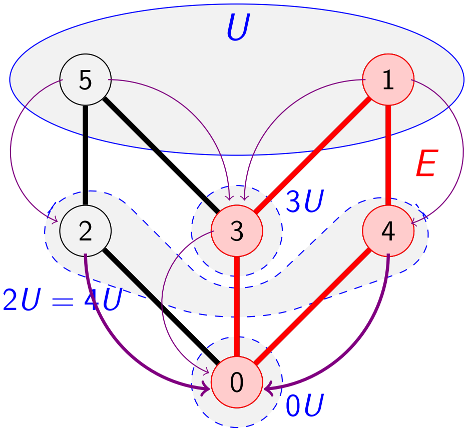

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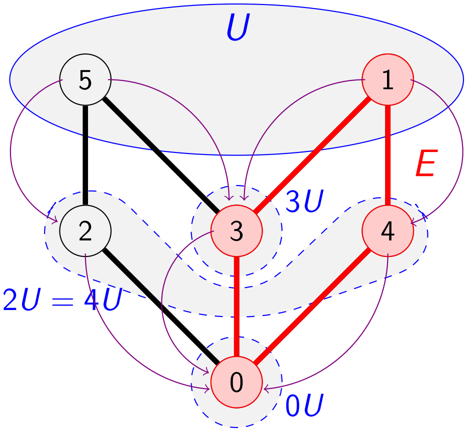

Outline

A construction in the opposite direction

- Let T be a meet semilattice.
- For all $s \in T$, let $\langle A_s, \cdot_s, *_s, \vee_s, e_s \rangle$ be disjoint partial algebras such that
  - $\langle A_s, \cdot_s, e_s \rangle$ are monoids,
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Proposition

If T a Boolean lattice, then A is a reduced Baer semigroup with

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Open problems

- What about Rickart rings which are not reduced?
    - Probably also C-rings have the same structure.
- Let $\langle R, +, \cdot, -, ', 0, 1\rangle$ be a reduced Rickart ring.
    - Recall that R admits a structure of strong semilattice of multiplicative skew nearlatttices $\{eU\}_{e\in E}$.
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Thank you for your attention

Questions?