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Reduced Rickart rings and skew nearlattices

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2 Introduction

- Reduced Rickart rings
- Skew nearlattices

3 Constructing the strong semilattice of semigroups

- Singular skew nearlattices in a reduced Rickart ring
- The strong semilattice of multiplicative skew nearlattices in a reduced Rickart ring

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4 Back to the ring?

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- Reduced Rickart rings
- Skew nearlattices
- 3 Constructing the strong semilattice of semigroups
 - Singular skew nearlattices in a reduced Rickart ring
 - The strong semilattice of multiplicative skew nearlattices in a reduced Rickart ring
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- Any m-domain ring can be decomposed into disjoint semigroups – see [Subrahmanyam 1960].
- It can be proved that reduced Rickart rings and m-domain rings are the same thing.
- Any reduced Rickart ring R has a structure of skew nearlattice – see [Cīrulis 2015].

Question

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What happens to the skew nearlattice structure when we decompose the ring into semigroups?

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Answers

• We get a structure of strong semilattice of semigroups.

- The semigroups are actually skew nearlattices.
- When we try to "reconstruct" a reduced Rickart ring from its strong semilattice of semigroups, we get a reduced Baer semigroup.

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Reduced Rickart rings					

Definition

A semigroup S is called Baer semigroup iff for every $a \in S$ there are idempotents $e, f \in S$ such that, for all $x \in S$,

• ax = 0 iff ex = x,

• xa = 0 iff xf = x.

Definition

A unitary ring *R* whose multiplicative semigroup is a Baer semigroup is called a Rickart ring.

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Reduced Rickart rings					

Definition

A ring is called reduced iff it has no non-zero nilpotent elements.

Proposition

On a reduced Rickart ring, the idempotents e and f from the definition are unique and coincide.

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Reduced Rickart rings					

Focal operation

Definition

For every a in a reduced Rickart ring R, let a' be the unique idempotent such that

$$ax = 0 \iff a'x = x$$

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for all x. The operation ' is called focal operation.

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Deduced Distant				

Examples of reduced Rickart rings

$\blacksquare \mathbb{Z}$

Any Boolean ring

\square \mathbb{Z}_{pq} for prime numbers $p \neq q$



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Reduced Rickart rings					

Abian order

Proposition

On a reduced ring, the following relation is a partial order:

$$a \leq b$$
 iff $ab = a^2$

Definition

It is called the Abian order.

 A reduced Rickart ring ordered by the Abian order is a semi-Boolean algebra. [Janowitz 1976]

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Example: The Abian order of \mathbb{Z}_6 and the focal operation



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Example: The Abian order of \mathbb{Z}_6 and the focal operation



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Example: The Abian order of \mathbb{Z}_6 and the focal operation



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Skew nearlattices				



Definition

A poset P is called nearlattice if

P is a meet-semilattice,

if $x, y \in P$ have an upper bound, then they have the join (i.e., P is finitely bounded complete).

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Skew nearlattice	e			

Definition

A semigroup is called band if all its elements are idempotent.

- Let $\langle S, \circ \rangle$ be a band.
- Let $x \leq_{\circ} y$ iff xy = x = yx.
- Then ≤₀ is a partial order, called the natural order of the band S.

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Skew nearlattices

Right skew nearlattices ([Cīrulis 2015])

Definition

A (right) skew nearlattice is a partial algebra $\langle S, *, \lor \rangle$ such that

- There is a finitely bounded complete order \leq on *S*
- ∨ is the respective partial join operation
- * is an associative operation
- $x \lor y = y$ if and only if x * y = x.
- The operation * is idempotent.
- The natural order on the band $\langle S, * \rangle$ is \leq .
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Skew nearlattices				

Let I, V be sets.

• Let $\mathcal{F}(\mathcal{I}, \mathcal{V})$ be the set of partial functions from I to V.

■ For partial functions *f*, *g*, define

$$f \stackrel{\leftarrow}{\cap} g := g|_{\operatorname{dom}(f) \cap \operatorname{dom}(g)}$$

- Then $\langle \mathcal{F}(\mathcal{I}, \mathcal{V}), \overleftarrow{\cap} \rangle$ is a band.
- The union of sets is their join with respect to the natural order of the band.
- $\langle \mathcal{F}(\mathcal{I}, \mathcal{V}), \overleftarrow{\cap}, \cup \rangle$ is a skew nearlattice.

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Skew pearlattices				

• Let *R* be a reduced Rickart ring.

Define the skew meet:

 $a \overleftarrow{\wedge} b := a'' b$

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• $\langle R, \overleftarrow{\wedge} \rangle$ is a band.

- Its natural order coincides with the Abian order.
- Let \lor be the respective partial join.
- Then $\langle R, \lor, \overleftarrow{\land} \rangle$ is a right skew nearlattice.

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$$\langle R, \overleftarrow{\wedge} \rangle$$
 is a band.

.

- Its natural order coincides with the Abian order.
- Let \lor be the respective partial join.

• Then
$$\langle R, \lor, \overleftarrow{\land} \rangle$$
 is a right skew nearlattice.

Motivation 00	Introduction	The strong semilattice of semigroups 00 00000	Back to the ring? 00	The end 000

Right zero bands and right singular skew nearlattices

Definition

• A band $\langle S, \circ \rangle$ is called right zero band if

$$x \circ y = y$$

for all $x, y \in S$.

■ A skew nearlattice (S,*, ∨) is called right singular if its band reduct (S,*) is right zero.

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Skew nearlattices				

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Outline

1 Motivation

2 Introduction

- Reduced Rickart rings
- Skew nearlattices

3 Constructing the strong semilattice of semigroups

- Singular skew nearlattices in a reduced Rickart ring
- The strong semilattice of multiplicative skew nearlattices in a reduced Rickart ring
- 4 Back to the ring?

5 The end

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Proposition

- Let R be a reduced Rickart ring.
- Let U be the set of non-zero divisors.
- Let e be an idempotent.

• Let
$$a \wedge b := a''b$$
.

• Then $\langle eU, \overleftarrow{\wedge}, \vee \rangle$ is a right singular skew nearlattice.

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The strong semilattice of semigroups 00

Singular skew nearlattices in a reduced Rickart ring

Example: The singular skew nearlattices $\langle eU, \overleftarrow{\wedge}, \lor angle$ in \mathbb{Z}_6

- Abian order of \mathbb{Z}_6
- Lattice of idempotents E
- The set of non-zero
- The other right



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Reduced Rickart rings and skew nearlattices

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Reduced Rickart rings and skew nearlattices

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The strong semilattice of multiplicative skew nearlattices in a reduced Rickart ring				

Definition

- Let $\langle A, *_A, \lor_A \rangle$ and $\langle B, *_B, \lor_B \rangle$ be skew nearlattices.
- A map $f : A \rightarrow B$ is called homomorphism of skew nearlattices if
 - $f : \langle A, *_A \rangle \longrightarrow \langle B, *_B \rangle$ is a semigroup homomorphism, ■ whenever $x, y \in A$ have the join, so do their images, and

 $f(x \vee y) = f(x) \vee f(y).$

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The strong semilattice of multiplicative skew nearlattices in a reduced Rickart ring					

Definition (Clifford)

- Let *T* be a meet-semilattice,
- Let $\{\langle A_s, *_s \rangle \mid s \in T\}$ be a family of disjoint semigroups,
- Suppose that, for all $s, t \in T$ with $s \leq t$, there are semigroup homomorphisms $f_s^t : A_t \to A_s$ such that
- f_t^t are the identity maps

• if
$$r \leq s \leq t$$
, then $f_r^s f_s^t = f_r^t$.

- Let $A := \bigcup_{s \in T} A_s$.
- For $x \in A_s, y \in A_t$ define $x \overleftarrow{\wedge} y \coloneqq f_{s \wedge t}^s(x) *_{s \wedge t} f_{s \wedge t}^t(y)$.
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- The algebra $\langle A, \overleftarrow{\wedge} \rangle$ is called a strong semilattice of the semigroups $\{A_s\}_{s \in T}$.

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The strong semilattice of multiplicative skew nearlattices in a reduced Rickart ring						

Multiplicative skew nearlattices

Definition

A partial algebra $\langle S,*,\vee,\cdot\rangle$ will be called multiplicative skew nearlattice if

- $\langle S, *, \lor \rangle$ is a skew nearlattice,
- $\langle S, \cdot \rangle$ is a monoid.

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The strong semilattice of multiplicative skew nearlattices in a reduced Rickart ring					

Theorem

- Let R be a reduced Rickart ring.
- Let U be the set of non-zero divisors.
- Let e be an idempotent.
- Let $a \overleftarrow{\wedge} b \coloneqq a'' b$.
- Then ⟨eU, ∕∧, ∨, ·⟩ is a multiplicative right singular skew nearlattice.
- Moreover, $\left\langle R,\overleftarrow{\wedge},\cdot\right
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- Then $\langle eU, \overleftarrow{\wedge}, \lor, \cdot \rangle$ is a multiplicative right singular skew nearlattice.
- Moreover, $\langle R, \overleftarrow{\wedge}, \cdot \rangle$ is a strong semilattice of the multiplicative skew nearlattices $\langle eU, \lor, \overleftarrow{\wedge}, \cdot \rangle$.

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Motivation		The strong semilattice of semigroups	Back to the ring?	The end	
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The strong semilattice of multiplicative skew nearlattices in a reduced Rickart ring					

- Abian order of \mathbb{Z}_6
- Lattice of idempotents E
- The set of non-zero divisors U
- The other right singular skew nearlatttices eU
- The homomorphisms of skew nearlattices



Reduced Rickart rings and skew nearlattices

Motivation		The strong semilattice of semigroups	Back to the ring?	The end	
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Motivation		The strong semilattice of semigroups	Back to the ring?	The end	
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Motivation		The strong semilattice of semigroups	Back to the ring?	The end
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Motivation		The strong semilattice of semigroups	Back to the ring?	The end
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The strong semi	lattice of multiplicative	skew nearlattices in a reduced Rickart ring		

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Reduced Rickart rings and skew nearlattices

Motivation 00	Introduction 000000 000000	The strong semilattice of semigroups 00 00000	Back to the ring? ○○	The end 000

Outline

1 Motivation

2 Introduction

- Reduced Rickart rings
- Skew nearlattices

3 Constructing the strong semilattice of semigroups

- Singular skew nearlattices in a reduced Rickart ring
- The strong semilattice of multiplicative skew nearlattices in a reduced Rickart ring

4 Back to the ring?

5 The end

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Motivation		The strong semilattice of semigroups	Back to the ring?	The end
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■ Let *T* be a meet semilattice.

- For all s ∈ T, let (A_s, ·_s, *_s, ∨_s, e_s) be disjoint partial algebras such that
 - $\langle A_s, \cdot_s, e_s \rangle$ are monoids,
 - $A_0 = \{e_0\},\$
 - $\langle A_s, *_s, \vee_s \rangle$ are right singular skew nearlattices,
 - there are homomorphisms $f_s^t : A_t \to A_s$ for all $s \leq t$,
 - these homomorphisms induce a strong semilattice of multiplicative skew nearlattices.

• Let $A = \bigcup_{s \in T} A_s$.

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• Let $A = \bigcup_{s \in T} A_s$.

Motivation		The strong semilattice of semigroups	Back to the ring?	The end
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- Let T be a meet semilattice.
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Motivation 00	Introduction 000000 000000	The strong semilattice of semigroups 00 00000	Back to the ring? ○●	The end 000

Back to the ring?

Proposition

If T a Boolean lattice, then A is a reduced Baer semigroup with $x \cdot y := f_{s \wedge t}^{s}(x) \cdot_{s \wedge t} f_{s \wedge t}^{t}(y),$ $0 := e_{0},$ $1 := e_{1}.$

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Motivation 00	Introduction 000000 000000	The strong semilattice of semigroups 00 00000	Back to the ring? ○●	The end 000

Back to the ring?

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Motivation 00	Introduction 000000 000000	The strong semilattice of semigroups 00 00000	Back to the ring? 00	The end

Outline

1 Motivation

2 Introduction

- Reduced Rickart rings
- Skew nearlattices

3 Constructing the strong semilattice of semigroups

- Singular skew nearlattices in a reduced Rickart ring
- The strong semilattice of multiplicative skew nearlattices in a reduced Rickart ring

4 Back to the ring?

5 The end

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Motivation 00	Introduction 000000 000000	The strong semilattice of semigroups 00 00000	Back to the ring? 00	The end ●00

What about Rickart rings which are not reduced?

Probably also C-rings have the same structure.

• Let $\langle R, +, \cdot, -, ', 0, 1 \rangle$ be a reduced Rickart ring.

- Recall that R admits a structure of strong semilattice of multiplicative skew nearlatttices {eU}_{ref}.
- We can construct a Baer semigroup from it.
- Is this Baer semigroup it isomorphic to the reduct $(R, \cdot, ', 0, 1)$?

Reduced Rickart rings and skew nearlattices

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Motivation 00	Introduction 000000 000000	The strong semilattice of semigroups 00 00000	Back to the ring? 00	The end ●00

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Motivation 00	Introduction 000000 000000	The strong semilattice of semigroups 00 00000	Back to the ring? 00	The end ○●○

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Demonstratio Mathematica 48(4), 2015, 493-508.

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Motivation		The strong semilattice of semigroups	Back to the ring?	The end
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References II



I. Cremer

On reduced Rickart rings (Latvian), Master thesis *University of Latvia, 2016*

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Thank you for your attention

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