Matthew effects via team semantics (work in progress)

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Joint work with S. Frittella[1], G. Greco[2], M. Piazzai[2,3] and N. Wijnberg[3]

1 University of Orléans, France 2 TU Delft
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The Matthew effect (Merton 1968)

For whoever has will be given more, and they will have an abundance.
Whoever does not have, even what they have will be taken from them.

Matthew, 25:29

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“The rich get richer; the poor get poorer.”

Matthew effects

- sociology of science: (Merton 1968)

- business:
- social network:

Matthew effects

- sociology of science: (Merton 1968)

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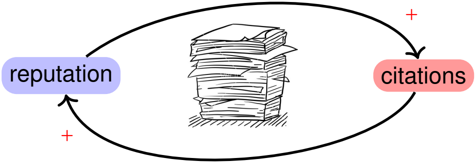

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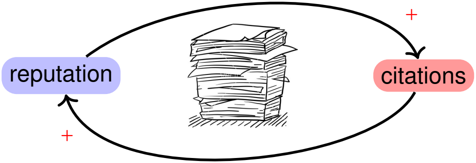


- business:

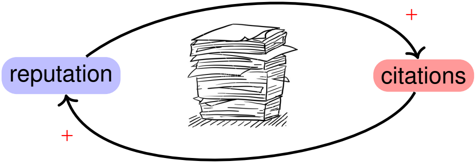
reviews

sales

- social network:

Matthew effects

- sociology of science: (Merton 1968)



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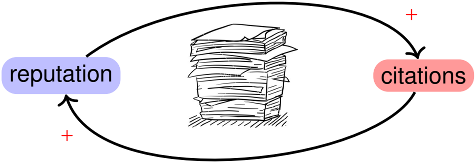

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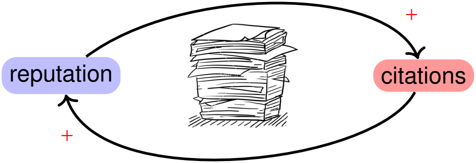

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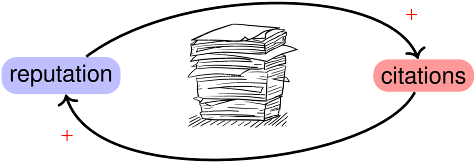

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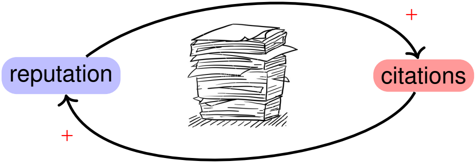

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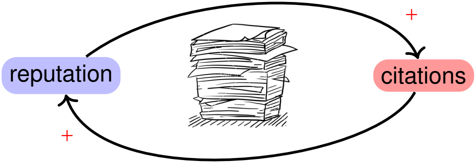

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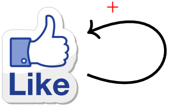

Data sets and regression analysis

Sales	Reviews	Time
0	2	2010
3	5	2011
6	8	2012
8	10	2013
9	14	2014
12	15	2015

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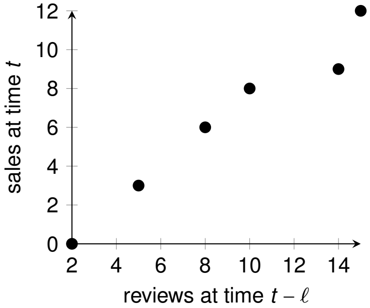

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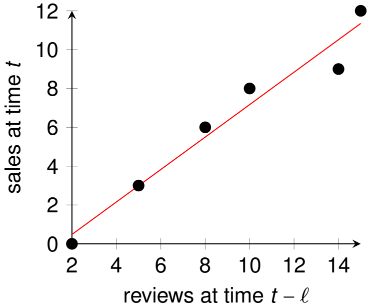

$$s_{(t)} = \beta_0 + \beta_1 r_{(t-\ell)} + \epsilon,$$

where $\beta_1 > 0$.

Dependence relation

Independent variables ($x_1 \ldots x_n$)

Dependent variable (y)

x_1	$\ldots$	x_n	w_1	w_2	y	t
1	$\ldots$	2			3	2000
3	$\ldots$	5	$\vdots$	$\vdots$	6	2001
6	$\ldots$	7			10	2002
7	$\ldots$	8	$\vdots$	$\vdots$	15	2003
12	$\ldots$	8			21	2004

$$y_{(t)} = \beta_0 + \beta_1(x_1)_{(t-\ell)} + \beta_2(x_2)_{(t-\ell)} + \cdots + \beta_n(x_n)_{(t-\ell)} + \epsilon$$

Dependence relation

Independent variables

Dependent variable

Time variable

x_1	$\dots$	x_n	w_1	w_2	y	t
1	$\dots$	2			3	2000
3	$\dots$	5	$\vdots$	$\vdots$	6	2001
6	$\dots$	7			10	2002
7	$\dots$	8	$\vdots$	$\vdots$	15	2003
12	$\dots$	8			21	2004

$$y_{(t)} = \beta_0 + \beta_1 (x_1)_{(t-\ell)} + \beta_2 (x_2)_{(t-\ell)} + \cdots + \beta_n (x_n)_{(t-\ell)} + \epsilon$$

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x_1	$\dots$	x_n	w_1	w_2	y	t
1	$\dots$	2			3	2000
3	$\dots$	5	$\vdots$	$\vdots$	6	2001
6	$\dots$	7			10	2002
7	$\dots$	8	$\vdots$	$\vdots$	15	2003
12	$\dots$	8			21	2004

$$
\begin{aligned}
y_{(t)} &= \beta_0 + \beta_1 (x_1)_{(t-\ell)} + \beta_2 (x_2)_{(t-\ell)} + \cdots + \beta_n (x_n)_{(t-\ell)} + \epsilon \\
y_{(t-\ell)} &= \beta_0 + \beta_1 (x_1)_{(t-2\ell)} + \beta_2 (x_2)_{(t-2\ell)} + \cdots + \beta_n (x_n)_{(t-2\ell)} + \epsilon
\end{aligned}
$$

$$
y_{(t)} - y_{(t-\ell)} = \beta_1 \Big((x_1)_{(t-\ell)} - (x_1)_{(t-2\ell)} \Big) + \epsilon
$$

If $\beta_1 > 0$:

$$
\begin{tikzcd}
 & y_t \\
x_{t-\ell} \arrow[ur, squiggly] & y_{t-\ell} \arrow[u] \\
x_{t-2\ell} \arrow[u] &
\end{tikzcd}
$$

Dependence relation

Independent variables (over $x_1 \ldots x_n$) — Dependent variable (y) — Time variable (t)

x_1	$\ldots$	x_n	w_1	w_2	y	t
1	$\ldots$	2			3	2000
3	$\ldots$	5	$\vdots$	$\vdots$	6	2001
6	$\ldots$	7			10	2002
7	$\ldots$	8	$\vdots$	$\vdots$	15	2003
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$$y_{(t)} = \beta_0 + \beta_1 (x_1)_{(t-\ell)} + \beta_2 (x_2)_{(t-\ell)} + \cdots + \beta_n (x_n)_{(t-\ell)} + \epsilon$$
$$y_{(t-\ell)} = \beta_0 + \beta_1 (x_1)_{(t-2\ell)} + \beta_2 (x_2)_{(t-2\ell)} + \cdots + \beta_n (x_n)_{(t-2\ell)} + \epsilon$$

$$y_{(t)} - y_{(t-\ell)} = \beta_1 \Big((x_1)_{(t-\ell)} - (x_1)_{(t-2\ell)} \Big) + \epsilon$$

If $\beta_1 > 0$:

$$\begin{tikzcd} & y_t \\ x_{t-\ell} \arrow[ur, squiggly] & y_{t-\ell} \arrow[u] \\ x_{t-2\ell} \arrow[u] & \end{tikzcd}$$

$x \nearrow_{\ell} y$

Team semantics & (In)dependence logic

$\vec{u}$	$\vec{w}$	x	y	t
		1	3	2000
$\vdots$	$\vdots$	3	6	2002
		6	10	2004
$\vdots$	$\vdots$	7	15	2006
		12	21	2008

- Team semantics (Hodges 1997)
- Dependence logic (Väänänen 2007): FO+ $=\!(t, y)$
- Independence logic (Grädel, Väänänen 2013): FO + $x \perp y$

Team semantics & (In)dependence logic

	$\vec{u}$	$\vec{w}$	x	y	t
			1	3	2000
	$\vdots$	$\vdots$	3	6	2002
assignment s			6	10	2004
	$\vdots$	$\vdots$	7	15	2006
			12	21	2008

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	$\vec{u}$	$\vec{w}$	x	y	t
			1	3	2000
	⋮	⋮	3	6	2002
assignment s			6	10	2004
	⋮	⋮	7	15	2006
$M \vDash_s x \nearrow_\ell y$?			12	21	2008

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Team semantics & (In)dependence logic

a set of assignments

$\vec{u}$	$\vec{w}$	x	y	t
		1	3	2000
$\vdots$	$\vdots$	3	6	2002
		6	10	2004
$\vdots$	$\vdots$	7	15	2006
		12	21	2008

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a team D:

a set of assignments

$M \models_D x \nearrow_{\ell} y$

$\vec{u}$	$\vec{w}$	x	y	t
		1	3	2000
$\vdots$	$\vdots$	3	6	2002
		6	10	2004
$\vdots$	$\vdots$	7	15	2006
		12	21	2008

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Team semantics & (In)dependence logic

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a set of assignments

$\vec{u}$	$\vec{w}$	x	y	t
		1	3	2000
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- Dependence logic (Väänänen 2007): $\mathrm{FO} + {=}(t, y)$
- Independence logic (Grädel, Väänänen 2013): $\mathrm{FO} + x \perp y$
- Inquisitive logic (Ciardelli, Groenendijk, Roelofsen 2011)

  [stay tuned,

  stay for the next two talks...]

a team D:
a set of assignments

$\vec{u}$	$\vec{w}$	x	y	t
		1	3	2000
$\vdots$	$\vdots$	3	6	2002
		6	10	2004
$\vdots$	$\vdots$	7	15	2006
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- Team semantics (Hodges 1997)
- Dependence logic (Väänänen 2007): FO+ $=\!(t, y)$ $\exists f$
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a set of assignments

$\vec{u}$	$\vec{w}$	x	y	t
		1	3	2000
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- Independence logic (Grädel, Väänänen 2013): FO + $x \perp y$
- $x \nearrow_{\ell} y$

  $$y_{(t)} = \alpha_0 + \beta x_{(t-\ell)} + \alpha_1 (w_1)_{(t-\ell)} + \cdots + \alpha_n (w_n)_{(t-\ell)} + \epsilon$$

Definition. $M \vDash_D x \nearrow_{\ell} y$ iff $\exists p(x, \vec{w}, y)$ as above with $\beta > 0$ s.t.
$M \vDash_D x \nearrow_{\ell}^{p} y$,

a team D:

a set of assignments

$\vec{u}$	$\vec{w}$	x	y	t	δ
		1	3	2000	2
$\vdots$	$\vdots$	3	6	2002	2
		6	10	2004	2
$\vdots$	$\vdots$	7	15	2006	2
		12	21	2008	2

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- Dependence logic (Väänänen 2007): FO+ $=\!(t, y)$ $\exists f$
- Independence logic (Grädel, Väänänen 2013): $\mathrm{FO} + x \perp y$
- $x \nearrow_\ell y$ term $\ell ::= \delta \mid k\delta$

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Team semantics & (In)dependence logic

a team D:

a set of assignments

$\vec{u}$	$\vec{w}$	x	y	t	δ	
		1	3	2000	2	
$\vdots$	$\vdots$	3	6	2002	2	s
		6	10	2004	2	s'
$\vdots$	$\vdots$	7	15	2006	2	
		12	21	2008	2	

- Team semantics (Hodges 1997)
- Dependence logic (Väänänen 2007): FO+ $=(t, y)$

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Definition. $M \vDash_D x \nearrow_\ell y$ iff $\exists p(x, \vec{w}, y)$ as above with $\beta > 0$ s.t. $M \vDash_D x \nearrow_\ell^{p} y$, namely, for all $s, s' \in D$,

$$s(t) = s'(t) + s'(\ell^M) \Longrightarrow s(y) \approx^M \beta s'(x) + q(s'(\vec{w})).$$

Dependence relation

- $M \vDash_{\mathrm{D}} x_1, \ldots, x_n \nearrow_{\ell}^{\mathrm{p}} y$ iff for all $s, s' \in D$,

  $s(t) = s'(t) + s'(\ell^M) \Longrightarrow s(y) \approx^M \beta_1 s'(x_1) + \cdots + \beta_n s'(x_n) + q(s'(\vec{w}))$,

  where p is the above polynomial and $\beta_1, \ldots, \beta_n > 0$.

- $M \vDash_{\mathrm{D}} x_1, \ldots, x_n \nearrow_{\ell} y$
  iff there exists $p(\vec{x}, \vec{w}, y)$ with $\beta_1, \ldots, \beta_n > 0$ s.t. $M \vDash_D \vec{x} \nearrow_{\ell}^{\mathrm{p}} y$

- $x_1, \ldots, x_n \searrow_{\ell} y$ is defined similarly except that $\beta_1, \ldots, \beta_n$ are required to be < 0.

Dependence relation

- $M \vDash_{\mathrm{D}} x_1, \ldots, x_n \nearrow_{\ell}^{\mathrm{p}} y$ iff for all $s, s' \in D$,

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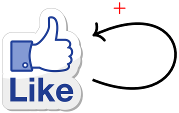
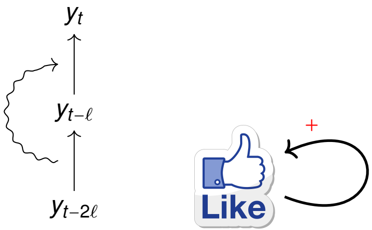


- y is subject to a (positive) direct ℓ-Matthew effect if
  $y_{(t)} = \alpha_0 + \beta y_{(t-\ell)} + \alpha_1(x_1)_{(t-\ell)} + \cdots + \alpha_n(x_n)_{(t-\ell)} + \epsilon$,
  where $\beta > 0$.
- Define $\mathrm{DME}_\ell y := y \nearrow_\ell y$.

Matthew effects



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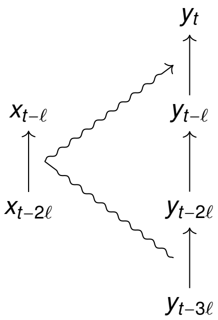
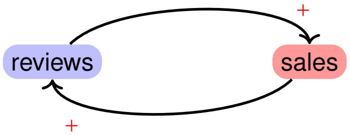


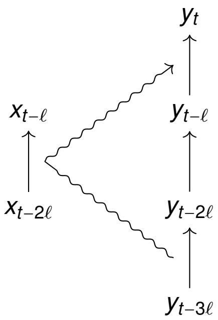


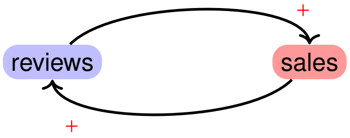
- y is subject to a (positive) x-mediated ℓ-Matthew effect if

$$\begin{aligned} y_{(t)} &= \alpha_0 + \beta(x)_{(t-\ell)} + \alpha_1(x_1)_{(t-\ell)} + \cdots + \beta_n(x_n)_{(t-\ell)} + \epsilon, \\ x_{(t)} &= \gamma_0 + \delta(y)_{(t-\ell)} + \gamma_1(x_1)_{(t-\ell)} + \cdots + \gamma_n(x_n)_{(t-\ell)} + \epsilon, \end{aligned}$$

  where $\beta, \delta > 0$.
- Define $\mathrm{MME}_\ell(y, x) := (x \leadsto_\ell y) \wedge (y \leadsto_\ell x)$
- $\mathrm{DME}_\ell y \vDash \mathrm{MME}_\ell(y, y)$

Matthew effects

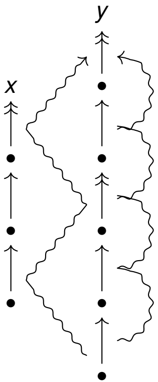


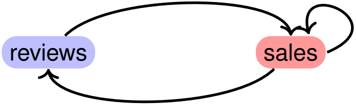


- y is subject to a (positive) x-mediated ℓ-Matthew effect if

  $$y_{(t)} = \alpha_0 + \beta(x)_{(t-\ell)} + \alpha_1(x_1)_{(t-\ell)} + \cdots + \beta_n(x_n)_{(t-\ell)} + \epsilon,$$
  $$x_{(t)} = \gamma_0 + \delta(y)_{(t-\ell)} + \gamma_1(x_1)_{(t-\ell)} + \cdots + \gamma_n(x_n)_{(t-\ell)} + \epsilon,$$

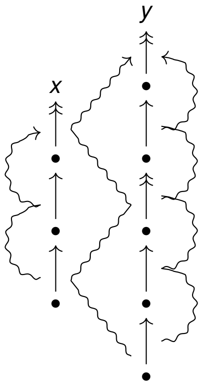
  where $\beta, \delta > 0$.
- Define $\mathrm{MME}_\ell(y, x) := (x \nearrow_\ell y) \wedge (y \nearrow_\ell x)$
- $\mathrm{DME}_\ell y \models \mathrm{MME}_\ell(y, y)$

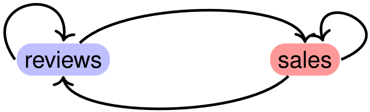




- y is subject to a (positive) comple ℓ-Matthew effect w.r.t. x:
  $\mathrm{CME}_\ell y(x) ::= \mathrm{MME}_\ell(y, x) \wedge \mathrm{DME}_\ell y$
- x and y are subject to a (positive) double comple ℓ-Matthew effect:
  $\mathrm{CME}_\ell(x, y) ::= \mathrm{MME}_\ell(y, x) \wedge \mathrm{DME}_\ell x \wedge \mathrm{DME}_\ell y$

Matthew effects





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  $\mathsf{CME}_\ell(x,y) ::= \mathsf{MME}_\ell(y,x) \wedge \mathsf{DME}_\ell x \wedge \mathsf{DME}_\ell y$

Properties of dependence relation

- (Commutativity) $x_1, \cdots, x_n \nearrow_\ell y \vDash x_{i_1}, \cdots, x_{i_n} \nearrow_\ell y$
- (Duplication) $x_1, \ldots, x_n \nearrow_\ell y \vDash x_i, x_1, \ldots, x_n \nearrow_\ell y$
- (Projection) $x_1, \ldots, x_n \nearrow_\ell y \vDash x_{i_1}, \ldots, x_{i_k} \nearrow_\ell y$, where $k \leq n$
- (Regrouping) $(\vec{x} \nearrow_\ell^{\mathrm{p}} y), (\vec{z} \nearrow_\ell^{\mathrm{p}} y) \vDash \vec{x}, \vec{z} \nearrow_\ell^{\mathrm{p}} y$
- (Transitivity) $(\vec{x} \nearrow_\ell y),\ (y \nearrow_{\ell'} z) \vDash \vec{x} \nearrow_{\ell+\ell'} z$
- (Enhancing) $x \nearrow_\ell x \vDash x \nearrow_{k\ell} x$
- (Reflexivity) $\vDash x \nearrow_0 x$

Properties of Matthew effects

- $\mathrm{MME}_{\ell}(x, y), \mathrm{MME}_{\ell}(y, z) \vDash \mathrm{MME}_{2\ell}(x, z)$, i.e., $(x \rightsquigarrow_{\ell} y), (y \rightsquigarrow_{\ell} x), (y \rightsquigarrow_{\ell} z), (z \rightsquigarrow_{\ell} y) \vDash (x \rightsquigarrow_{2\ell} z) \wedge (z \rightsquigarrow_{2\ell} x)$
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$$
\begin{tikzcd}
 & y_t \\
x_{t-\ell} & y_{t-\ell} \arrow[u, ""{name=C}] \\
x_{t-2\ell} \arrow[u, ""{name=B}] & y_{t-2\ell} \arrow[u] \\
 & y_{t-3\ell} \arrow[u, ""{name=A}]
\arrow[from=A, to=B, squiggly, dash]
\arrow[from=B, to=C, squiggly]
\end{tikzcd}
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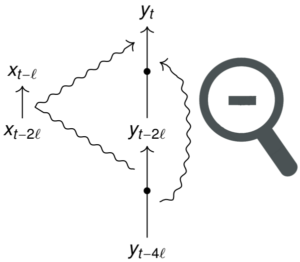
Properties of Matthew effects



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A logic of Matthew effects (ML)

Syntax

$\phi ::= \alpha \mid \neg\alpha \mid \vec{x} \nearrow_\ell y \mid \vec{x} \searrow_\ell y \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \otimes \phi \mid \exists x \phi \mid \forall x \phi$

Team semantics

- $M \vDash_D \alpha$ iff for all $s \in D$, $M \vDash_s \alpha$
- $M \vDash_D \neg\alpha$ iff for all $s \in D$, $M \nvDash_s \alpha$
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- $M \vDash_D \phi \otimes \psi$ iff there exist $D_0, D_1 \subseteq D$ with $D = D_0 \cup D_1$ s.t.

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	5	2	2
...	10	4	3
	12	15	4
	16	21	5

$(x < 10)$

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$$(x < 10) \\ \otimes \\ \mathrm{MME}_\ell(x, y)$$

Comparison with dependence and independence logic

- (Väänänen, Kontinen 2009) Dependence Logic (D $:=$ FO$+ =\!(\vec{x}, y)$) can express all existential second-order downward closed properties.
- (Galliani 2012) Independence Logic (I $:=$ FO $+ \vec{x} \perp \vec{y}$) can express all existential second-order properties.
- ML $\leq$ D $<$ I, i.e., for every ML-formula ϕ, there is D-formula $\tau(\phi)$ s.t.

$$M \models_D \phi \iff M \models_D \tau(\phi).$$

- There is a deduction system (via translation into independence logic) such that

$$\Gamma \models \phi \iff \Gamma \vdash \phi,$$

where ϕ is $\otimes$-free and has no quantification over $\vec{x} \rightsquigarrow_{\ell} y$. (follows from (Hannula 2013)&(Y. 2016))

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Future work

- (Full) axiomatization of ML without going through the translation.
- Comparison with other dependency notions.
- To study the notion of $\vec{x} \nearrow^{\mathrm{r}}_{\ell} y$, where r represents a regression that has been (actually) performed on the dataset in question.

  There is (indeed) a difference between $\vec{x} \nearrow^{\mathrm{p}}_{\ell} y$ and $\vec{x} \nearrow^{\mathrm{r}}_{\ell} y$, even if r generates the same regression function for y as p.

- Different levels of abstraction: $\vec{x} \nearrow_{\ell} y$, $\vec{x} \nearrow^{\mathrm{p}}_{\ell} y$ and $\vec{x} \nearrow^{\mathrm{r}}_{\ell} y$
- To consider other parameters in a Matthew effect.
  E.g., the strength of a Matthew effect (which roughly corresponds to the β in $y_{(t)} = \beta(y)_{(t-\ell)} + q(\vec{w})$).

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