Constructive canonicity for lattice-based fixed point logics

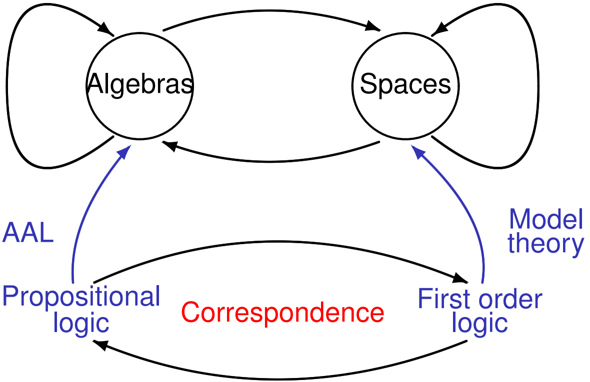
Zhiguang Zhao
Joint work with
Willem Conradie, Andrew Craig and Alessandra Palmigiano

TACL 2017

Unified correspondence

Generalised Sahlqvist theory

From a model theoretic problem to an algebraic logic problem



Unified correspondence

- DLE-logics [CP12, CPS]
- substructural logics [CP]
- hybrid logics [CR15]
- many valued modal logics [CIRM]
- mu calculus [CFPS15, CGP14, CC15]
- regular modal logics [PSZ16]
- possibility semantics [YZ]
- Jónsson-style vs Sambin-style canonicity [PSZ15]
- constructive canonicity [CP]
- Sahlqvist via translation [CPZ]
- constructive canonicity for lattice-based fixed point logics [CCPZ]
- display calculi [GMPTZ16]
- sequent calculi [MZ16]
- finite lattices and monotone modal logic [FPS16]

What is constructive canonicity?

Preservation of validity of inequalities under (constructive) canonical extensions:

$$\mathbb{A} \models \varphi \leq \psi \quad \Rightarrow \quad \mathbb{A}^\delta \models \varphi \leq \psi.$$

Constructive canonical extension of lattice $\mathbb{A}$ (c.f. Gehrke-Harding 2001)

Complete lattice $\mathbb{A}^\delta$ containing $\mathbb{A}$ as a dense and compact sublattice

In the presence of the Axiom of Choice, $\mathbb{A}^\delta$ is perfect:

- $J^\infty(\mathbb{A}^\delta)$ is completely join-dense in $\mathbb{A}^\delta$, and
- $M^\infty(\mathbb{A}^\delta)$ is completely meet-dense in $\mathbb{A}^\delta$.

In the constructive setting: not enough join/meet-irreducibles

Our results

[Conradie Craig 2014]: canonicity for mu-calculus

- distributive-based, with fixed points, specific signature
- non-constructive metatheory

[Conradie Palmigiano]: constructive canonicity

- general lattice-based, no fixed points, arbitrary signature
- constructive metatheory

[CCPZ]: constructive canonicity for lattice-based fixed point logics

- general lattice-based, with fixed points, arbitrary signature
- constructive metatheory
- simpler ALBA! No specific rules for fixed points

A general strategy of canonicity via ALBA

$$\begin{array}{ccc}
\mathbb{A} \models \alpha \leq \beta & & \mathbb{A}^\delta \models \alpha \leq \beta \\
\Updownarrow & & \\
\mathbb{A}^\delta \models_{\mathbb{A}} \alpha \leq \beta & & \Updownarrow \\
\Updownarrow & & \\
\mathbb{A}^\delta \models_{\mathbb{A}} \mathrm{ALBA}(\alpha \leq \beta) & \Longleftrightarrow & \mathbb{A}^\delta \models \mathrm{ALBA}(\alpha \leq \beta)
\end{array}$$

We apply this strategy to lattice-based logics with fixed points

Two interpretations of fixed point operators

Motivation: completeness

Problem: canonical extension changes the values of fixed point formulas

In the lattice expansion $\mathbb{A}$:

$$\mu x.t(x, a_1, \ldots, a_{n-1}) := \bigwedge\{\, a \in A \mid t(a, a_1, \ldots, a_{n-1}) \leq a \,\}$$

if this meet exists, otherwise $\mu x.t(x, a_1, \ldots, a_{n-1})$ is undefined.

In the canonical extension $\mathbb{A}^\delta$ of lattice expansion $\mathbb{A}$:

$$\mu^* x.t(x, a_1, \ldots, a_{n-1}) := \bigwedge\{\, a \in A \mid t(a, a_1, \ldots, a_{n-1}) \leq a \,\}$$

Consequence: two definitions of canonicity

Two definitions of canonicity

$\varphi \leq \psi$ is canonical:

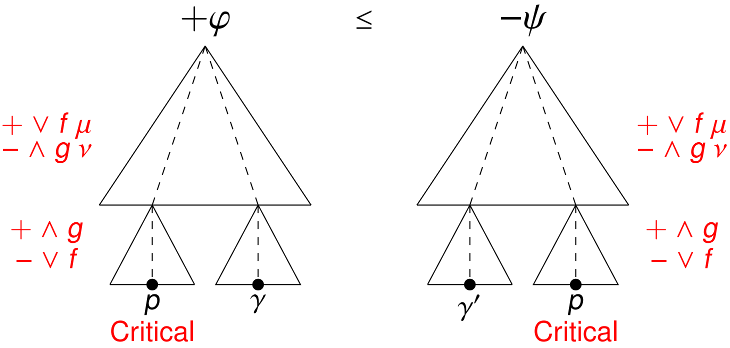
$$\mathbb{A} \models \varphi \leq \psi \quad \Rightarrow \quad \mathbb{A}^\delta \models \varphi \leq \psi.$$

$\varphi \leq \psi$ is tame canonical:

$$\mathbb{A} \models \varphi \leq \psi \quad \Rightarrow \quad \mathbb{A}^\delta \models \varphi^* \leq \psi^*.$$

From the two notions of canonicity, two syntactic characterizations arise of formulas guaranteed to be canonical for each type:

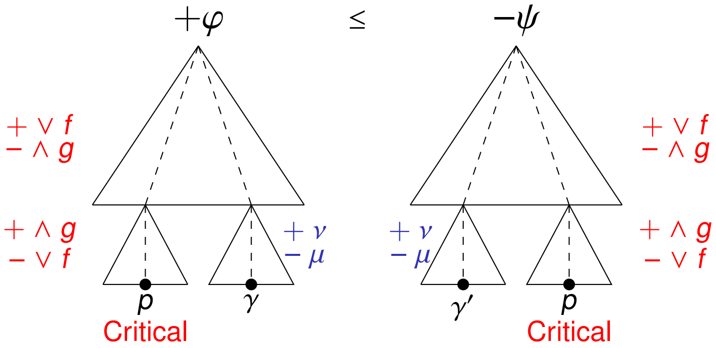
Canonicity



Two Syntactic Characterizations

From the two notions of canonicity, two syntactic characterizations arise of formulas guaranteed to be canonical for each type:

Tame canonicity



Further directions

- Fixed points modelling different forms of group knowledge in the context of the epistemic logic of categories
- Use canonicity to prove conservativity of proof systems

References

[Conradie Craig] Canonicity results for mu-calculi: an algorithmic approach, *JLC*, 2017.

[Conradie Fomatati Palmigiano Sourabh] Correspondence theory for intuitionistic modal mu-calculus, *TCS*, 564:30-62 (2015).

[Conradie Ghilardi Palmigiano] Unified Correspondence, in *Johan van Benthem on Logic and Information Dynamics*, Springer, 2014.

[Conradie Palmigiano 2012] Algorithmic Correspondence and Canonicity for Distributive Modal Logic, *APAL*, 163:338-376.

[Conradie Palmigiano 2015] Algorithmic correspondence and canonicity for non-distributive logics, submitted.

[Conradie Palmigiano 2015] Algorithmic correspondence and canonicity for non-distributive logics, submitted. [Conradie Palmigiano 2015] Constructive canonicity of inductive inequalities, submitted.

[Conradie Palmigiano Sourabh] Algebraic modal correspondence: Sahlqvist and beyond, JLAMP, 2016.

[Conradie Palmigiano Zhao] Sahlqvist via Translation, submitted, 2016.

[Conradie Robinson] On Sahlqvist Theory for Hybrid Logics, *JLC*, 2017.

[Frittella Palmigiano Santocanale] Dual characterizations for finite lattices via correspondence theory for monotone modal logic, *JLC*, 2017.

[Greco Ma Palmigiano Tzimoulis Zhao] Unified correspondence as a proof-theoretic tool, *JLC*, 2016.

[Palmigiano Sourabh Zhao(a)] Sahlqvist theory for impossible worlds, *JLC*, 2017.