Algorithmic correspondence and canonicity for possibility semantics

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Possibility semantics

Possibility semantics

- a variant of standard Kripke semantics for modal logic
- motivation: partial possibilities vs total worlds
- constructive study of classical (modal) logic:
  - intuitionistic-style semantics: refinement relation
  - constructive completeness proofs
  - relation to constructive canonical extension

Existing works

- Duality: Holliday 2016
- Correspondence: Yamamoto 2016
- Canonicity: Holliday 2016

Possibility semantics

- possibility frame: $\mathbb{F} = (W, R, \sqsubseteq, \mathrm{RO}(W, \sqsubseteq))$
- possibility model: $\mathbb{M} = (\mathbb{F}, V)$ where $V : \mathrm{Prop} \to \mathrm{RO}(W, \sqsubseteq)$
- refinement relation $\sqsubseteq$: partial order on W
- accessibility relation R: binary relation on W
- $\mathrm{RO}(W, \sqsubseteq)$: set of admissible valuations
- intuition behind $\mathrm{RO}(W, \sqsubseteq)$: subsets equal to their "double negation"

Possibility semantics

Satisfaction relation

- $\mathbb{F}, V, w \vDash p$ iff $w \in V(p)$;
- $\mathbb{F}, V, w \vDash \varphi \wedge \psi$ iff $\mathbb{F}, V, w \vDash \varphi$ and $\mathbb{F}, V, w \vDash \psi$;
- $\mathbb{F}, V, w \vDash \varphi \vee \psi$ iff $(\forall v \sqsubseteq w)(\exists u \sqsubseteq v)(\mathbb{F}, V, u \vDash \varphi$ or $\mathbb{F}, V, u \vDash \psi)$;
- $\mathbb{F}, V, w \vDash \varphi \rightarrow \psi$ iff $(\forall v \sqsubseteq w)(\mathbb{F}, V, v \vDash \varphi \Rightarrow \mathbb{F}, V, v \vDash \psi)$;
- $\mathbb{F}, V, w \vDash \neg\varphi$ iff $(\forall v \sqsubseteq w)(\mathbb{F}, V, v \nvDash \varphi)$;
- $\mathbb{F}, V, w \vDash \Box\varphi$ iff $\forall v(Rwv \Rightarrow \mathbb{F}, V, v \vDash \varphi)$.

Algebraic correspondence: from frames to algebras

$$
\begin{array}{ccc}
\mathbb{B} \models \forall \vec{p}(\varphi(\vec{p})) & \Leftrightarrow & \mathbb{F} \Vdash \varphi(\vec{p}) \\
\Updownarrow & & \\
\mathbb{B} \models \forall \vec{\mathbf{i}}\mathrm{Pure}(\varphi(\vec{p})) & \Leftrightarrow & \mathbb{F} \Vdash \mathrm{FO}(\mathrm{Pure}(\varphi(\vec{p})))
\end{array}
$$

- In the dual BAO of Kripke frames, nominals are interpreted as atoms.
- How about possibility semantics?

Dual algebras

Given $\mathbb{F} = (W, \sqsubseteq, R, \mathsf{RO}(W, \sqsubseteq))$, the regular open dual BAO $\mathbb{B}_{\mathsf{RO}}$

- $\mathbb{B}_{\mathsf{RO}}$ is a complete and completely additive BAO, but not necessarily atomic.
- lack of atomicity: what is the consequence in correspondence theory?

Nominals and their interpretations

Algebraic setting	Interpretation for nominals	Dually corresponding to
perfect Boolean algebras	atoms	singletons
perfect distributive lattices	complete join-primes	$w\uparrow$
perfect general lattices	complete join-irreducibles	Galois closure of singletons
constructive canonical extensions	closed elements	N.A.
complex algebras of possibility frames		regular open closures of singletons

Our results

- Correspondence results for inductive formulas over full possibility frames
- Correspondence results for inductive formulas over filter-descriptive possibility frames
- Constructive canonicity-via-correspondence