Undefinability of standard sequent calculi for Paraconsistent logics

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Topology, Algebra and Categories in Logic.
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Outline

1. Kleene Logics
2. The starting point: PWK
3. Deductive calculi for paraconsistent Kleene logics
4. Our results

Kleene tables

Strong Kleene tables:

$\wedge$	0	$\frac{1}{2}$	1
0	0	0	0
$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$
1	0	$\frac{1}{2}$	1

$\vee$	0	$\frac{1}{2}$	1
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- Paraconsistent Weak Kleene logic, PWK: $\langle \mathbf{WK}, \{1, \frac{1}{2}\} \rangle$

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$$\Gamma \vDash_{\mathbf{PWK}} \alpha \iff \text{for every } v, \quad v[\Gamma] \subseteq \{1, 1/2\} \Rightarrow v(\alpha) \in \{1, 1/2\}$$

A closer look to **WK**

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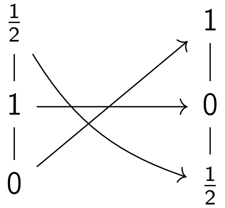

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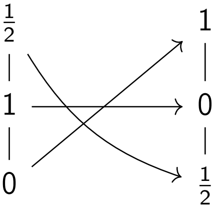

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Counterexample to absorption:

$$1 \wedge (1 \vee \frac{1}{2}) = \frac{1}{2} \neq 1$$

Involutive bisemilattices

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- I6 $x \wedge (\neg x \vee y) \approx x \wedge y$;
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Theorem

$\mathbb{V}(\mathbf{WK}) = \mathcal{IBSL}$.

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L is algebraizable if it has an equivalent algebraic semantics.

PWK, $\mathcal{IBSL}$, and Algebraic Logic

Theorem

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PWK is not selfextensional, i.e. the interderivability relation $\dashv\vdash_{\mathrm{L}}$ is not a congruence on **Fm**.

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2. LP: Hilbert style (Font), sequent calculi (Avron, Beall)

Sequent calculus for PWK

Axioms

$$\alpha \Rightarrow \alpha$$

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Structural rules

$$\frac{\Gamma \Rightarrow \Delta}{\Gamma, \alpha \Rightarrow \Delta}\ \mathit{LW} \qquad \frac{\Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \alpha, \Delta}\ \mathit{RW}$$

$$\frac{\Gamma \Rightarrow \Delta, \alpha \qquad \Gamma, \alpha \Rightarrow \Delta}{\Gamma \Rightarrow \Delta}\ \mathit{Cut}$$

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Operational rules

$$\frac{\Gamma \Rightarrow \alpha, \Delta}{\Gamma, \neg\alpha \Rightarrow \Delta}\ L\neg \qquad \frac{\Gamma, \alpha \Rightarrow \Delta}{\Gamma \Rightarrow \neg\alpha, \Delta}\ R\neg$$

$$\frac{\Gamma, \alpha \Rightarrow \Delta \qquad \Gamma, \beta \Rightarrow \Delta}{\Gamma, \alpha \vee \beta \Rightarrow \Delta}\ L\vee \qquad \frac{\Gamma \Rightarrow \alpha, \beta, \Delta}{\Gamma \Rightarrow \alpha \vee \beta, \Delta}\ R\vee$$

Proviso for $L\neg$: $\mathrm{var}(\alpha) \subseteq \mathrm{var}(\Delta)$

Sequent calculi for LP

Avron (2014).
Axioms and structural rules as for PWK, logical rules including:

$$\frac{\Gamma, \neg\alpha \Rightarrow \Delta \qquad \Gamma, \neg\beta \Rightarrow \Delta}{\Gamma, \neg(\alpha \wedge \beta) \Rightarrow \Delta}\, L\neg\wedge \qquad \frac{\Gamma, \alpha \Rightarrow \Delta}{\Gamma, \neg\neg\alpha \Rightarrow \Delta}\, L\neg\neg$$

Our starting question

Both LP and PWK have sequent calculi with some "limitations" on operational rules.

Is it possible to give standard sequent calculi for LP and PWK?

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5. Only classical structural rules, i.e. contraction, weakening and cut are (possibly) allowed.

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$\mathbb{IFL}$ is the family of logics in the language $\{\neg, \wedge, \vee\}$ s.t.:

1. Have the same theorems of classical logic.
2. The connective $\neg$ has a fixed point on a designated value and $k \approx \neg\neg k$.

$\mathbb{IFL}$: basic facts

Remark

If $\mathrm{L} \in \mathbb{IFL}$ then L is paraconsistent, i.e. $\alpha, \neg\alpha \nvdash_{\mathrm{L}} \beta$

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Lemma

A logic $\mathrm{L} \in \mathbb{IFL}$ can not have a non designated value k such that $k = \neg k$.

Standard Gentzen Calculi

In a standard calculus ($L\wedge$) and ($R\vee$) must be of the form:

$$\frac{\Gamma, \alpha, \beta \Rightarrow \Delta}{\Gamma, \alpha \wedge \beta \Rightarrow \Delta}\, L\wedge \qquad \frac{\Gamma \Rightarrow \alpha, \beta, \Delta}{\Gamma \Rightarrow \alpha \vee \beta, \Delta}\, R\vee$$

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That is

$$\frac{P_1, \quad ..., \qquad \Gamma, \alpha \Rightarrow \Delta \quad ..., \qquad P_n}{\Gamma \Rightarrow \neg\alpha, \Delta}$$

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Therefore $\mathcal{S}$ shall derive the sequent $\Rightarrow \neg(\alpha \wedge \neg\alpha) \vee \beta$.

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$$\dfrac{\dfrac{\dfrac{\alpha, \neg\alpha \Rightarrow \beta}{\alpha \wedge \neg\alpha \Rightarrow \beta}\ R\wedge}{\Rightarrow \neg(\alpha \wedge \neg\alpha), \beta}\ R\neg\ \text{(Lemma)}}{\Rightarrow \neg(\alpha \wedge \neg\alpha) \vee \beta}\ R\vee$$

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$$\dfrac{\dfrac{\dfrac{\alpha, \neg\alpha \Rightarrow \beta}{\alpha \wedge \neg\alpha \Rightarrow \beta}\, R\wedge}{\Rightarrow \neg(\alpha \wedge \neg\alpha), \beta}\, R\neg \text{ (Lemma)}}{\Rightarrow \neg(\alpha \wedge \neg\alpha) \vee \beta}\, R\vee$$

Since $\mathcal{S}$ is sound, the above derivation tree cannot terminate with axioms.

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4. The regularization of a logic.

Thank you!