First-order cologic for profinite structures

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This fact is key to the semantics of first-order logic, which describes structures by how they are built as directed colimits of finite pieces.

Dually, profinite structures are built as codirected limits of finite pieces.

Question: What's the right analogue of first-order logic, or "cologic", for describing profinite structures?

Generalizing first-order logic - The plan

In attempting to answer this question, we'll take a more abstract approach.

The plan:

- Given a locally finitely presentable category $\mathcal{D}$, define the notions of $\mathcal{D}$-*signature* Σ and Σ-*structure*: an object of $\mathcal{D}$ with extra algebraic and relational structure.
- Define the logic $\mathrm{FO}(\mathcal{D}, \Sigma)$ of Σ-structures in $\mathcal{D}$.
  - Explain how $\mathrm{FO}(\mathcal{D}, \Sigma)$ can be interpreted in an ordinary multi-sorted first-order setting.
- Given a category $\mathcal{D}$ whose dual is locally finitely presentable (e.g. a category of profinite structures), the *cologic* for Σ-*costructures* (objects of $\mathcal{D}$ with extra coalgebraic and corelational structure) is $\mathrm{FO}(\mathcal{D}^{\mathrm{op}}, \Sigma)$.
  - Describe applications/connections with other work.

LFP categories

Recall:

- An object c in a category $\mathcal{D}$ is *finitely presentable* if the functor $\mathrm{Hom}_{\mathcal{D}}(c, -)$ preserves directed colimits.
- (Gabriel & Ulmer) A category $\mathcal{D}$ is *locally finitely presentable* if:
  - It is cocomplete.
  - Every object is a directed colimit of finitely presentable objects.
  - The full subcategory $\mathcal{C}$ of finitely presentable objects is essentially small, i.e. there is a small full subcategory $\mathcal{A}$ containing a representative of every isomorphism class in $\mathcal{C}$.

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We fix an LFP category $\mathcal{D}$ and choose a category of representatives $\mathcal{A}$. We call:

- objects of $\mathcal{D}$ *domains*.
- objects of $\mathcal{C}$ *variable contexts*.
- objects of $\mathcal{A}$ *arities*.

The classical case: $\mathcal{D} = \mathrm{Set}$, $\mathcal{C} = \mathrm{FinSet}$, $\mathcal{A} = \omega$.

LFP categories - Examples

More LFP categories:

- Set^X, for any set X; $\mathcal{D}^{\mathcal{B}}$, for any LFP $\mathcal{D}$ and small category $\mathcal{B}$.
- Str_L, the category of L-structures; Grp; Ring; Poset; Cat; Mod_T, where T is a first-order universal Horn theory.
- $\mathrm{Lex}(\mathcal{C}^{\mathrm{op}}, \mathsf{Set})$, the finite-limit preserving presheaves on $\mathcal{C}$, for any small category $\mathcal{C}$ with finite colimits.
- ind$-\mathcal{C}$, the free cocompletion of $\mathcal{C}$ under directed colimits, for any small category $\mathcal{C}$ with finite colimits.

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Categories whose duals are LFP:

- pro$-\mathcal{C}$, the free completion of $\mathcal{C}$ under codirected limits, for any small category $\mathcal{C}$ with finite limits.
- ProFinSet $\cong$ Stone $\cong$ $\mathsf{Bool}^{\mathrm{op}}$; ProFinGrp.

Signatures and structures

Definition

A $\mathcal{D}$-signature Σ consists of, for every arity $n \in \mathcal{A}$,

- *A set $\mathfrak{R}_n$, called the n-ary relation symbols.*
- *An object $\mathfrak{F}_n \in D$, called the n-ary operations.*

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Given a $\mathcal{D}$-signature Σ, a Σ-structure is an object M in $\mathcal{D}$, together with, for every arity $n \in \mathcal{A}$,

- *A map of sets $\mathfrak{R}_n \to \mathcal{P}(\mathrm{Hom}_{\mathcal{D}}(n, M))$. The image of an n-ary relation symbol R is an "n-ary relation" $R^M \subseteq \mathrm{Hom}_{\mathcal{D}}(n, M)$.*
- *(Kelly & Power) A map of sets $\mathrm{Hom}_{\mathcal{D}}(n, M) \to \mathrm{Hom}_{\mathcal{D}}(\mathfrak{F}_n, D)$. The image of an n-tuple $a\colon n \to M$ is a map $\widehat{a}\colon \mathfrak{F}_n \to M$.*

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The classical case: An n-ary relation is a subset of $\mathrm{Hom}_{\mathsf{Set}}(n, M) \cong M^n$. If $\mathfrak{F}_n$ is the *set* of n-ary function symbols, and a an n-tuple, $\widehat{a}(f) = f(a)$.

Zero

It is a fact that $\mathcal{C}$ is always closed under finite colimits in $\mathcal{D}$. In particular, $\mathcal{A}$ contains an initial object, 0:

$$\mathrm{Hom}_{\mathcal{D}}(0, M) = \{*\}.$$

$\mathfrak{F}_0 =$ the constants. The map $\widehat{*}\colon \mathfrak{F}_0 \to M$ is the interpretation of the constants.

$\mathfrak{R}_0 =$ the proposition symbols. The interpretetation of a proposition symbol $R^M \subseteq \{*\}$ is either "true" (inhabited) or "false" (empty).

The term algebra

Σ a signature with algebraic part $\mathfrak{F} = (\mathfrak{F}_n)_{n \in \mathcal{A}}$.
(Adámek, Milius, & Moss) A Σ-structure can be viewed as an algebra for the *polynomial functor* $H_{\mathfrak{F}} \colon \mathcal{D} \to \mathcal{D}$, defined by

$$H_{\mathfrak{F}}(M) = \coprod_{n \in \mathcal{A}} \coprod_{\mathrm{Hom}_{\mathcal{D}}(n,M)} \mathfrak{F}_n$$

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Definition

Let $x \in \mathcal{C}$ and $n \in \mathcal{A}$. An n-term in context x is an arrow $n \to T(x)$.

By analyzing the structure of $T(x)$, it is possible to give a more concrete syntax for terms (depending on the category $\mathcal{D}$).

Evaluation

Given:

- M an Σ-structure.
- $t\colon n \to T(x)$ an n-term in context x.
- $a\colon x \to M$ an "interpretation of the variables in context x."

We obtain a map $t^M(a)$, the "evaluation of t in M".
Just use the universal property of $T(x)$ and compose:

$$\begin{tikzcd}
n \arrow[r, "t"'] \arrow[rr, bend left, "t^M(a)"] & T(x) \arrow[r, dashed] & M \\
 & x \arrow[u, "i"'] \arrow[ur, "a"'] &
\end{tikzcd}$$

Syntax

Definition

Let $x \in \mathcal{C}$ be a context. An atomic formula in context x is one of the following:

- $s(x) = t(x)$, *where s and t are n-terms in context x, for some $n \in \mathcal{A}$.*
- $R(t(x))$, *where t is an n-term in context x and R is a n-ary relation symbol, for some $n \in \mathcal{A}$.*

Definition

A formula in context x is one of the following:

- *An atomic formula in context x.*
- $\top(x)$, $\bot(x)$, $\neg\varphi(x)$, $\varphi(x) \wedge \psi(x)$, *or* $\varphi(x) \vee \psi(x)$, *where $\varphi(x)$ and $\psi(x)$ are formulas in context x.*
- $\exists(y|_f = x)\varphi(y)$ *or* $\forall(y|_f = x)\varphi(y)$, *where $f\colon x \to y$ is an arrow and $\varphi(y)$ is a formula in context y.*

Semantics

We now define the satisfaction relation $\models$. Given a Σ-structure M, a formula $\varphi(x)$ in context x, and $a\colon x \to M$:

- $M \models s(a) = t(a)$ iff $s^M(a) = t^M(a)$ in $\mathrm{Hom}_{\mathcal{D}}(n, M)$.
- $M \models R(t(a))$ iff $t^M(a) \in R^M \subseteq \mathrm{Hom}_{\mathcal{D}}(n, M)$.
- The Boolean connectives have their usual meaning.
- $M \models \exists(y|_f = x)\varphi(y)$ iff there exists $b\colon y \to M$ such that $b \circ f = a$ and $M \models \varphi(b)$.
- $M \models \forall(y|_f = x)\varphi(y)$ iff for all $b\colon y \to M$ such that $b \circ f = a$, $M \models \varphi(b)$.

$$\begin{tikzcd} y \arrow[r, dashed, "\exists/\forall\, b"] & M \\ x \arrow[u, "f"] \arrow[ur, "a"'] & \end{tikzcd}$$

Zero again

Definition

A sentence is a formula in the initial context 0.

We write t instead of $t(0)$ for terms in the initial context, φ instead of $\varphi(0)$ for sentences, and $\exists x\, \varphi(x)$ instead of $\exists(x|_* = *)\varphi(x)$ (and similarly for the universal quantifier).

When φ is a sentence, there is a unique interpretation of the variables $*\colon 0 \to M$, and we write $M \models \varphi$ instead of $M \models \varphi(*)$.

Examples

Let $\mathcal{D} = \mathrm{Cat}$.

- Let $x = \bullet \longrightarrow \bullet$ be the category with a single non-identity arrow.
- Let $y = \bullet \rightleftarrows \bullet$ be the category with a single inverse pair of non-identity arrows.
- Let i_1 and i_2 be the two functors $x \to y$ sending the non-identity arrow in x to each of the non-identity arrows of y.
- Let P be an x-ary relation. The interpretation of P in a category $\mathcal{A}$ picks out a set of maps $x \to A$, i.e. a set of arrows in $\mathcal{A}$.

What properties do the following sentences express?

$$\forall x \, \exists (y|_{i_1} = x) \top (y)$$

$$\forall x \, (P(x) \vee \exists (y|_{i_1} = x) P(t_{i_2}(y)))$$

Presheaf structures

Gabriel-Ulmer duality tells us that for any LFP category $\mathcal{D}$, $\mathcal{D} \cong \mathrm{Lex}(\mathcal{A}^{\mathrm{op}}, \mathsf{Set})$, the category of finite limit preserving functors $\mathcal{A}^{\mathrm{op}} \to \mathsf{Set}$, with the equivalence given by $M \mapsto \mathrm{Hom}_{\mathcal{D}}(-, M)$.

Presheaf structures

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Let $\mathrm{PSh}(\Sigma)$ be the ordinary first-order language consisting of:

- A sort S_n for each $n \in \mathcal{A}$.
- A function symbol $\widetilde{f}$ of sort $S_n \to S_m$ for each arrow $f\colon m \to n$.
- A relation symbol $\widetilde{R}$ of sort S_n for each n-ary relation symbol R.
- A function symbol t^* of sort $S_n \to S_m$ for each m-term in context n, $t\colon m \to T(n)$.

Given a Σ-structure M, let $\mathrm{PSh}(M)$ be the $\mathrm{PSh}(\Sigma)$-structure in which $S_n = \mathrm{Hom}_{\mathcal{D}}(n, M)$, $\widetilde{f}(a) = a \circ f$, $\widetilde{R} = R^M$, and $t^*(a) = t^M(a)$.

The first-order translation

Let $T_{\mathrm{PSh}(\Sigma)}$ be the first-order theory asserting:

- $n \mapsto S_n$ is a functor $\mathcal{A}^{\mathrm{op}} \to$ Set (i.e. $\widetilde{f \circ g} = \widetilde{g} \circ \widetilde{f}$ and $\widetilde{\mathrm{id}} = \mathrm{id}$).
- This functor preserves limits.
- Coherence conditions for the term functions t^*.

Theorem

The Gabriel-Ulmer equivalence lifts to an equivalence between the category of Σ-structures and the category of models of $T_{\mathrm{PSh}(\Sigma)}$.

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The Gabriel-Ulmer equivalence lifts to an equivalence between the category of Σ-structures and the category of models of $T_{\mathrm{PSh}(\Sigma)}$.

Theorem

There is a bitranslation between $\mathrm{FO}(\mathcal{D}, \Sigma)$-formulas $\varphi(n)$ in context $n \in \mathcal{A}$ and first-order $\mathrm{PSh}(\Sigma)$-formulas $\widehat{\varphi(x_n)}$ in a single variable of sort S_n, such that for any Σ-structure M and any $a\colon n \to M$,

$$M \models \varphi(a) \iff \mathrm{PSh}(M) \models \widehat{\varphi(a)}.$$

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Theorem

There is a bitranslation between $\mathrm{FO}(\mathcal{D},\Sigma)$*-formulas* $\varphi(n)$ *in context* $n \in \mathcal{A}$ *and first-order* $\mathrm{PSh}(\Sigma)$*-formulas* $\widehat{\varphi(x_n)}$ *in a single variable of sort* S_n*, such that for any* Σ*-structure* M *and any* $a\colon n \to M$*,*

$$M \models \varphi(a) \iff \mathrm{PSh}(M) \models \widehat{\varphi(a)}.$$

Also:

- Every $\mathrm{FO}(\mathcal{D},\Sigma)$ formula in context x is equivalent to one in some context $n \in \mathcal{A}$ (after picking an isomorphism $x \cong n$).
- Every first-order $\mathrm{PSh}(\Sigma)$-formula in variables of sorts $S_{n_1}, \ldots, S_{n_k}$ is equivalent to one in a single variable of sort $S_{n_1 \coprod \cdots \coprod n_k}$

So $\mathrm{FO}(\mathcal{D},\Sigma)$ is exactly as expressive as an ordinary first-order language, and we can import theorems (compactness, Löwenheim-Skolem, etc.) and definitions (stability, NIP, etc.) from first-order model theory for free.

Ultraproducts

Let U be an ultrafilter on I, and let $(M_i)_{i\in I}$ be an I-indexed family of Σ-structures. The "categorical ultraproduct"

$$\mathrm{Ult}_{\mathcal{D}}((M_i)_{i\in I}, U) = \varinjlim_{X\in U} \prod_{i\in X} M_i$$

can be made into a Σ-structure in a natural way.
[When $\mathcal{D} = \mathsf{Set}$, this agrees with the usual definition as long as all M_i are nonempty.]

Ultraproducts commute with the presheaf translation, so Łoś's Theorem automatically holds:

$$\mathrm{PSh}(\mathrm{Ult}_{\mathcal{D}}((M_i)_{i\in I}, U)) = \mathrm{Ult}_{\mathcal{D}}((\mathrm{PSh}(M_i)_{i\in I}), U)$$

This essentially comes down to the fact that for all finitely presentable n,

$$\mathrm{Hom}_{\mathcal{D}}(n, \mathrm{Ult}_{\mathcal{D}}((M_i)_{i\in I})) \cong \mathrm{Ult}_{\mathsf{Set}}((\mathrm{Hom}_{\mathcal{D}}(n, M_i))_{i\in I})$$

Algebras for finitary functors

- A functor $F \colon \mathcal{D} \to \mathcal{D}$ is *finitary* if it preserves directed colimits.
- (Adámek, Milius, and Moss) Any finitary functor F on an LFP category has a *presentation* as a quotient of a polynomial functor H by a set of “flat equations”. An F-algebra is the same thing as an H-algebra which “satisfies these equations”:

$$\begin{tikzcd} n \arrow[rr, shift left, "\eta"] \arrow[rr, shift right, "\eta'"'] & & H(m) \end{tikzcd} \qquad \begin{tikzcd} n \arrow[rr, shift left, "\eta"] \arrow[rr, shift right, "\eta'"'] \arrow[drrr] & & H(m) \arrow[r, "H(a)"] & H(M) \arrow[d, "\alpha"] \\ & & m \arrow[r, "\forall a"'] & M \end{tikzcd}$$

$$\begin{tikzcd} n \arrow[r, "\eta"'] \arrow[rr, bend left, "t_\eta"] & H(m) \arrow[r, "i"'] & T(m) \end{tikzcd}$$

- In our context “flat equations” are literally equational sentences:

$$\forall m(t_\eta(m) = t_{\eta'}(m))$$

- So $\mathrm{FO}(\mathcal{D}, \Sigma)$ can be applied to algebras for any finitary functor.

Cologic

Let's finally return to profinite structures. If $\mathcal{D}^{\mathrm{op}}$ is LFP, what does $\mathrm{FO}(\mathcal{D}^{\mathrm{op}}, \Sigma)$ say about $\mathcal{D}$?

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Structures are *coalgebras* for *cofinitary* functors on $\mathcal{D}$. "Corelations" and "coformulas" express properties of "cotuples" $M \to n$.

Example: $\mathcal{D} =$ Stone, n a finite discrete space, $M \to n$ is a partition of M into n clopen sets. Quantifiers quantify over refinements of a given partition.

$$\begin{tikzcd} M \arrow[r, dashed, "\exists/\forall"] \arrow[dr] & m \arrow[d] \\ & n \end{tikzcd}$$

Cologic - connections to other work

- "Universal coalgebra" (Rutten). Coalgebras for cofinitary functors on Stone are of some interest (see Kupke, Kurz, and Venema, e.g.). For example, coalgebras for the Vietoris functor are exactly descriptive general frames.
- Cherlin, van den Dries, and Macintyre introduced a "cologic" of profinite groups (e.g. Galois groups) in order to study the model theory of PAC fields. Their description of "cologic" is essentially the same as the first-order translation (via presheaf structures) of $\mathrm{FO}(\text{ProFinGrp}^{\mathrm{op}}, \emptyset)$.
- Projective Fraïssé theory (Irwin & Solecki). A corelational language Σ on Stone spaces is the ideal context for dualized Fraïssé theory (Panagiotopoulos). The $\mathrm{FO}(\text{Stone}^{\mathrm{op}}, \Sigma)$-theories of their limits are characterized by "$\aleph_0$-categoricity" and quantifier elimination.

Future Work

1. Broaden the scope by dropping some of the finitary hypotheses which ensure equivalence to first-order logic (and compactness). What is the relationship between the logic of coalgebras on Set and those on Stone (via the Stone-Čech compactification)?
2. In concrete profinite structures, both the tuples and cotuples are interesting. Is there a nice logic which talks about both?
3. Study model theoretic properties: nontrivial $\mathrm{FO}(\mathsf{Stone}^{\mathrm{op}}, \Sigma)$-theories *always* have the strict order property and the independence property (these are bad), but $\mathrm{FO}(\mathsf{ProFinGrp}^{\mathrm{op}}, \Sigma)$-theories can be model-theoretically tame. What's the deeper reason for this?

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