LOGIC OF RESOURCES AND CAPABILITIES

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Motivation

- **Organizations** are social units of agents structured and managed to meet a need, or pursue collective goals.
- **Competitive advantage** lends itself to be explained terms of *agency*, *knowledge*, *goals*, *capabilities* and *inter-agent coordination*.
- **Resource-based view:** Central role in determining the success of an organization is played by the acquisition, management, and transformation of *resources* within that organization.

Main features

- **STIT-logic** approach of *capabilities* captured via modalities and use of *resources* and their manipulation to provide a concrete handle on the notion of capabilities.
- **Constructive** character guarantees that each theorem translates into an effective procedure.

Language

$$\alpha ::= a \in \mathsf{AtRes} \mid 1 \mid 0 \mid \alpha \cdot \alpha \mid \alpha \sqcup \alpha \mid \alpha \sqcap \alpha,$$

$$A ::= p \in \mathsf{AtProp} \mid \top \mid \bot \mid A \vee A \mid A \wedge A \mid A \rightarrow A \mid \alpha \triangleright A \mid \Diamond A \mid \mathord{\Diamond\mkern-11mu\mid}\,\alpha \mid \alpha \unrhd \alpha.$$

- $\Diamond A$: *'the agent is able to bring about state of affairs A'*
- $\mathord{\Diamond\mkern-11mu\mid}\,\alpha$: *'the agent is in possession of resource α'*
- $\alpha \triangleright A$: *'whenever resource α is in possession of the agent, using α the agent is capable to bring about A'*
- $\alpha \unrhd \beta$: *'the agent is capable of getting β from α, whenever in possession of α'*

The logic of resources and capabilities

Axiom schemas for $\Diamond$ and $\Diamond\!\!\!\mid\,$

D1. $\Diamond(A \vee B) \leftrightarrow \Diamond A \vee \Diamond B$

D2. $\Diamond\bot \leftrightarrow \bot$

D3. $\Diamond\!\!\!\mid\,(\alpha \sqcup \beta) \leftrightarrow \Diamond\!\!\!\mid\,\alpha \vee \Diamond\!\!\!\mid\,\beta$

D4. $\Diamond\!\!\!\mid\,0 \leftrightarrow \bot$

Axiom schemas for $\rhd$ and $\unrhd$

B1. $(\alpha \sqcup \beta) \rhd A \leftrightarrow \alpha \rhd A \wedge \beta \rhd A$

B2. $0 \rhd A$

B3. $\alpha \rhd \beta \rhd A \rightarrow \alpha \cdot \beta \rhd A$

B4. $(\alpha \sqcup \beta) \unrhd \gamma \leftrightarrow \alpha \unrhd \gamma \wedge \beta \unrhd \gamma$

B5. $0 \unrhd \alpha$

B6. $\alpha \unrhd (\beta \sqcap \gamma) \leftrightarrow \alpha \unrhd \beta \wedge \alpha \unrhd \gamma$

B7. $\alpha \unrhd 1$

Interaction axiom schemas

BD1. $\Diamond\!\!\!\mid\,\alpha \wedge \alpha \rhd A \rightarrow \Diamond A$

BD2. $\alpha \unrhd \beta \rightarrow \alpha \rhd \Diamond\!\!\!\mid\,\beta$

The logic of resources and capabilities

Pure-resource entailments schemas

R1. $\sqcup$ and $\sqcap$ are commutative, associative, idempotent, and distribute over each other;

R2. $\cdot$ is associative with unit 1;

R3. $\alpha \vdash 1$ and $0 \vdash \alpha$

R4. $\alpha \cdot (\beta \sqcup \gamma) \vdash (\alpha \cdot \beta) \sqcup (\alpha \cdot \gamma)$ and $(\beta \sqcup \gamma) \cdot \alpha \vdash (\beta \cdot \alpha) \sqcup (\gamma \cdot \alpha)$.

and closed under modus ponens, uniform substitution and the following rules:

$$\frac{\alpha \vdash \beta}{\alpha \cdot \gamma \vdash \beta \cdot \gamma} \qquad \frac{A \vdash B}{\alpha \rhd A \vdash \alpha \rhd B} \qquad \frac{A \vdash B}{\Diamond A \vdash \Diamond B} \qquad \frac{\alpha \vdash \beta}{\gamma \mathrel{|\!\!\rhd} \alpha \vdash \gamma \mathrel{|\!\!\rhd} \beta}$$

$$\frac{\alpha \vdash \beta}{\gamma \cdot \alpha \vdash \gamma \cdot \beta} \qquad \frac{\alpha \vdash \beta}{\beta \rhd A \vdash \alpha \rhd A} \qquad \frac{\alpha \vdash \beta}{\Diamond\!\!\!\!\mid\;\, \alpha \vdash \Diamond\!\!\!\!\mid\;\, \beta} \qquad \frac{\alpha \vdash \beta}{\beta \mathrel{|\!\!\rhd} \gamma \vdash \alpha \mathrel{|\!\!\rhd} \gamma}$$

Completeness, canonicity and disjunction property

Heterogeneous LRC-*algebras* are tuples of the form

$$F = (\mathbb{A}, \mathbb{Q}, \rhd, \Diamond, \rhd\!\!\!\!-, \Diamond\!\!\!\!\!\mid\;)$$

where

- $\mathbb{A}$ is a Heyting algebra,
- $\mathbb{Q} = (Q, \sqcup, \sqcap, \cdot, 0, 1)$ is a bounded distributive lattice with binary join-preserving operator $\cdot$ with unit 1.
- $\rhd : \mathbb{Q} \times \mathbb{A} \to \mathbb{A}$, $\Diamond : \mathbb{A} \to \mathbb{A}$, $\rhd\!\!\!\!- : \mathbb{Q} \times \mathbb{Q} \to \mathbb{A}$, $\Diamond\!\!\!\!\!\mid\; : \mathbb{Q} \to \mathbb{A}$.
- Lindenbaum-Tarski argument guarantees completeness
- Standard argument guarantees disjunction property

Theorem

The axioms of LRC are canonical. Hence, for every heterogenerous LRC-algebra F, its canonical extension F^δ is a perfect LRC-algebra. Hence, the logic LRC is complete w.r.t. the class of perfect LRC-algebras.

Display-style calculus

Structural and operational symbols for pure Res-connectives:

Str.	$\odot$		$,$		$\gtrdot$		$\lessdot$		$\sqsupset$		$(\sqsubset)$	
Op.	$\cdot$		$\sqcap$	$\sqcup$		$(.\backslash)$		$(/.)$	$({}_{\sqcup}\backslash)$	$({}_{\sqcap}\backslash)$	$(/_{\sqcup})$	$(/_{\sqcap})$

Structural and operational symbols for the modal operators:

Str.	$\circ$		$\rhd$		$\obar$		$\rhd\!\!\!-$	
Op.	$\Diamond$			$\triangleright$	$\Diamond\!\!\!\mid$			$\triangleright\!\!\!-$

Structural and operational symbols for the adjoints and residuals of the modal operators:

Str.	$\bullet$		$\blacktriangleright$		$\bullet\!\!\mid$		$\blacktriangle\!\!\!\mid$		$\blacktriangleright\!\!\!-$	
Op.		$(\blacksquare)$		$(\blacktriangleright)$		$(\blacksquare\!\mid\!)$	$(\blacktriangle\!\!\!\mid)$			$(\blacktriangleright\!\!\!-)$

Introduction rules

$$\frac{\Gamma \vdash \alpha \qquad A \vdash X}{\alpha \triangleright A \vdash \Gamma \mathrel{\large\triangleright} X} \triangleright_L \qquad \frac{X \vdash \alpha \mathrel{\large\triangleright} A}{X \vdash \alpha \triangleright A} \triangleright_R$$

$$\frac{\Gamma \vdash \alpha \qquad \beta \vdash \Delta}{\alpha \mathrel{\triangleright\mkern-14mu-} \alpha \vdash \Gamma \mathrel{\large\triangleright\mkern-18mu-} \Delta} \mathrel{\triangleright\mkern-14mu-}_L \qquad \frac{\Gamma \vdash \alpha \mathrel{\large\triangleright\mkern-18mu-} \alpha}{\Gamma \vdash \alpha \mathrel{\triangleright\mkern-14mu-} \alpha} \mathrel{\triangleright\mkern-14mu-}_R$$

Rules corresponding to axioms

$$\frac{\Gamma \vdash (Y \mathrel{\blacktriangleright\mkern-12mu-} \Delta)\,,\,(Z \mathrel{\blacktriangleright\mkern-12mu-} \Delta)}{\Gamma \vdash (Y\,;Z) \mathrel{\blacktriangleright\mkern-12mu-} \Delta}\ \text{B4} \qquad \frac{\Gamma \vdash (Y \blacktriangleright W)\,,\,(Z \blacktriangleright W)}{\Gamma \vdash (Y\,;Z) \blacktriangleright W}\ \text{B1}$$

$$\text{B6}\ \frac{(\Gamma \mathrel{\blacktriangle\mkern-3mu\blacktriangle} X)\,,\,(\Gamma \mathrel{\blacktriangle\mkern-3mu\blacktriangle} Y) \vdash \Delta}{\Gamma \mathrel{\blacktriangle\mkern-3mu\blacktriangle} (X\,;Y) \vdash \Delta} \qquad \frac{X \vdash \Gamma \vartriangleright (\Delta \vartriangleright Y)}{X \vdash \Gamma \odot \Delta \vartriangleright Y}\ \text{B3}$$

$$\frac{X \vdash \Gamma \vartriangleright \bullet Y}{X \vdash \text{⦶}\Gamma > Y}\ \text{BD1} \qquad \frac{X \vdash \Gamma \mathrel{\vartriangleright\mkern-12mu-} \text{◐}\, Y}{X \vdash \Gamma \vartriangleright Y}\ \text{BD2}$$

Display rules … but

$$\frac{\circ X \vdash Y}{X \vdash \bullet Y} \qquad \frac{\obar \Gamma \vdash X}{\Gamma \vdash \LEFTcircle X} \qquad \frac{X \vdash \Gamma \rhd Y}{\Gamma \vdash X \blacktriangleright Y} \qquad \frac{X \vdash \Gamma \mathrel{\rhd\!\!\!-} \Delta}{\Gamma \blacktriangle X \vdash \Delta} \qquad \frac{X \vdash \Gamma \mathrel{\rhd\!\!\!-} \Delta}{\Gamma \vdash X \mathrel{\blacktriangleright\!\!\!-} \Delta}$$

Notice the argument of the second coordinate of $\triangleright$ cannot be displayed

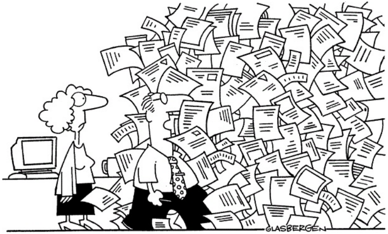
Cut rules

$$\frac{(X \vdash Y)[A]^{succ} \qquad A \vdash Z}{(X \vdash Y)[Z/A]^{succ}} \qquad \frac{\Gamma \vdash \alpha \qquad \alpha \vdash \Delta}{\Gamma \vdash \Delta}$$

Canonical Cut-elimination and subformula property

Follow from a general meta-theorem.

Homework correction

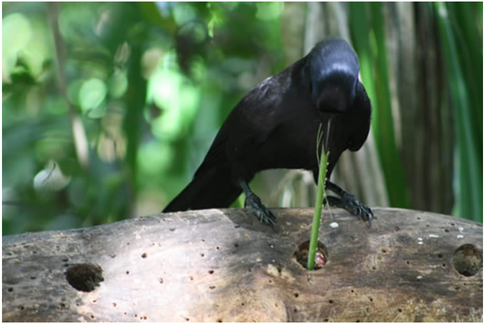


"I have some paperwork to catch up. If I'm not back in two days, organize a search and rescue team!"

Capabilities		initial state	planning
$\alpha \rhd_{\mathtt{c}} P_\alpha$	$\beta \rhd_{\mathtt{c}} P_\beta$	$\Diamond\!\!\!\mid_{\mathtt{c}} \alpha$	$M_\beta \to \Diamond\!\!\!\mid_{\mathtt{c}} \beta$
$\alpha \rhd_{\mathtt{d}} M_\alpha$	$\beta \rhd_{\mathtt{d}} M_\beta$	$\Diamond\!\!\!\mid_{\mathtt{d}} \beta$	$P_\alpha \to \Diamond\!\!\!\mid_{\mathtt{d}} \alpha$

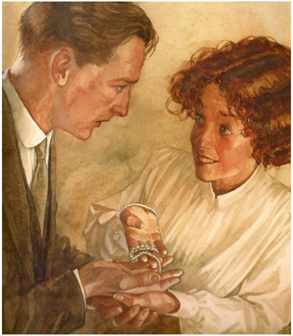
$$\mathsf{Ex}_{\mathtt{i}} \ \frac{X \vdash Y}{\circ_{\mathtt{i}} X \vdash Y}$$

THE WISDOM OF THE CROW



$$\frac{\Sigma \odot \Sigma \vdash \Omega}{\Sigma \vdash \Omega} \qquad \frac{(\Gamma \blacktriangle X) \odot (\Pi \blacktriangle Y) \vdash \Delta}{(\Gamma \odot \Pi) \blacktriangle (X \,;\, Y) \vdash \Delta} \qquad \frac{(\Gamma \blacktriangle X) \blacktriangle Y \vdash \Delta}{\Gamma \blacktriangle (X \,;\, Y) \vdash \Delta}$$

The gift of the magi



$$\Diamond\!\!\!\mid^1\sigma \wedge \Diamond\!\!\!\mid^2\xi \wedge [\sigma, \xi] \rhd \chi \to \Diamond\Diamond\!\!\!\mid^2\chi,$$

which is equivalent on perfect LRC-algebras to the following analytic rule:

$$\frac{\circ\Phi^2[\Sigma, \Xi] \blacktriangle X \vdash Y}{\Phi^1\Sigma\,;\Phi^2\Xi\,; X \vdash Y}\ \mathrm{RR}$$