# ALBA-style Sahlqvist preservation for modal compact Hausdorff spaces

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Canonicity, i.e. the preservation of validity of formulas from descriptive general frames to their underlying Kripke frames, is an important notion in modal logic, since it provides a uniform strategy for proving the strong completeness of axiomatic extensions of a basic (normal modal) logic. Thanks to its importance, the notion of canonicity has been explored also for other non-classical logics. In [19], Jónsson gave a purely algebraic reformulation of the frame-theoretic notion of canonicity, which he defined as the preservation of validity under taking canonical extensions, and proved the canonicity of Sahlqvist identities in a purely algebraic way. The construction of canonical extension was introduced by Jónsson and Tarski [20] as a purely algebraic encoding of the Stone spaces dual to Boolean algebras. In particular, the denseness requirement directly relates to the zero-dimensionality of Stone spaces. A natural question is then for which classes of formulas do canonicity-type preservation results hold in topological settings in which compactness is maintained and zero-dimensionality is generalized to the Hausdorff separation condition. This question has been addressed in [1, 2]. Specifically, in [1], Bezhanishvili, Bezhanishvili and Harding gave a canonicity-type preservation result for Sahlqvist formulas from modal compact Hausdorff spaces to their underlying Kripke frames, and in [2], Bezhanishvili and Sourabh generalized this result to modal fixed point formulas.

The proposed talk reports on an ongoing work in which the canonicity-type preservation results in [1, 2] are reformulated in a purely algebraic way in the spirit of Bjarni Jónsson. This work pertains to the wider theory of *unified correspondence*, which aims at identifying the underlying principles of Sahlqvist-type canonicity and correspondence for non-classical logics. As explained in [11, 7], this theory is grounded on the Stone-type dualities between the algebraic and the relational semantics of non-classical logics, and explains the "Sahlqvist phenomenon" in terms of the order-theoretic properties of the algebraic interpretations of the connectives of a non-classical logic. The focus on these properties has been crucial to the possibility of generalizing the Sahlqvist-type results from modal logic to a wide array of non-classical logics, including intuitionistic and distributive and general (non-distributive) lattice-based (modal) logics [8, 10, 6], non-normal (regular) modal logics [25], monotone modal logic [15], hybrid logics [14], many valued logics [22] and bi-intuitionistic and lattice-based modal mu-calculus [3, 4, 5]. In addition, unified correspondence has effectively provided overarching techniques unifying different methods for proving both canonicity and correspondence: in [24], the methodology pioneered by Jónsson [19] and the one pioneered by Sambin-Vaccaro [26] were unified; in [9, 12], constructive canonicity proposed by Ghilardi and Meloni [17] was unified with the Sambin-Vaccaro methodology; in [13], the Sambin-Vaccaro correspondence has been unified with the methodology of correspondence via translation introduced by Gehrke, Nagahashi and Venema in [16]. Recently, a very surprising connection has been established between the notions and techniques developed in unified correspondence and structural proof theory, which made it possible to solve a problem, opened by Kracht [21], concerning the characterization of the axioms which can be transformed into analytic structural rules [18, 23].

The main tools of unified correspondence are a purely order-theoretic definition of *inductive formulas/inequalities*, and the algorithm ALBA, which computes the first-order correspondent of input formulas/inequalities and is guaranteed to succeed on the inductive class.

In this talk, we illustrate how the preservation result of [1, 2] can be encompassed into unified correspondence theory. Intermediate steps toward this goal are: the identification of the order-theoretic properties which guarantee the Esakia lemma to hold, the proof of a suitably adapted version of the topological Ackermann lemma, and the introduction of the version of the algorithm ALBA appropriate for the "compact Hausdorff" setting.

Together, these results show that the same proof techniques introduced by Jónsson to prove the canonicity of Sahlqvist identities directly fuel the canonicity-type preservation of Sahlqvist formulas in the setting of modal compact Hausdorff spaces. I will also discuss further generalizations, and more in general a systematic approach to canonicity-type preservation results to which these preliminary results pave the way.

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