Properties of the annhilator of fuzzy subgroups

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1 Introduction

Duality theory for locally compact abelian groups was initially studied by Pontryagin [3]. This duality lies at the core of Fourier transform techniques in abstract harmonic analysis. We can describe Pontryagin's approach in the following way: Take first the circle group of the complex plane $\mathbb{T}$ endowed with its natural topology, as dualizing object. Then assign to a group X in the class of locally compact abelian groups the group $X^\wedge := \mathrm{CHom}(X;\mathbb{T})$ of continuous homomorphisms, and endow it with the compact open topology. This is precisely the dual group of X. After observing that the dual of a discrete group is compact and conversely, Pontryagin proved that the dual of a locally compact abelian group X is again a locally compact abelian group. This operation can be done a second time and then we obtain the second dual group $X^{\wedge\wedge}$ which, as it is known today, is topologically isomorphic to the initial locally compact group X.

In this context the notion of annihilator of a subgroup plays an important role. If $G \subset X$ is a subgroup of X, its *annihilator* is defined as the subgroup $G^\perp := \{\varphi \in X^\wedge : \varphi(G) = \{1\}\}$. If L is a subgroup of $X^\wedge$, the *inverse annihilator* is defined by ${}^\perp L := \{g \in X : \varphi(g) = 1, \forall \varphi \in L\}$. One important reason to study annihilators is the following: If G is a closed subgroup of a locally compact abelian group then ${}^\perp(G^\perp) = G$ [2].

2 Results

We introduce an extension of the notion of annihilator of a subgroup to the more general framework of fuzzy subgroups. Observe that constant functions $\underline{\lambda}$ are elementary examples of fuzzy subgroups. This shows that the class of fuzzy subgroups of a group is much larger than the class of subgroups.

As it can be noted, the definition of annihilator and inverse annihilator depends only of the group of continuous characters of the group X. In case of discrete subgroups, since $\mathrm{CHom}(X;\mathbb{T})$ coincides exactly with $\mathrm{Hom}(X;\mathbb{T})$ it is a purely algebraic notion. We develop a notion of annihilator of a fuzzy subgroup in this context. Our definition relies on the use of the α-levels of the fuzzy subgroup [1].

We will show that the formula ${}^\perp(G^\perp) = G$ is also true in the fuzzy framework. Then we will present another properties of the annihilator related with unions and intersections and we will conclude with some non trivial examples [4] were our definition can be applied .

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