Constructive canonicity for lattice-based fixed point logics

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The present contribution lies at the crossroads of at least three active lines of research in nonclassical logics: the one investigating the semantic and proof-theoretic environment of fixed point expansions of logics algebraically captured by varieties of (distributive) lattice expansions [1, 19, 24, 2, 16]; the one investigating constructive canonicity for intuitionistic and substructural logics [17, 25]; the one uniformly extending the state-of-the-art in Sahlqvist theory to families of nonclassical logics, and applying it to issues both semantic and proof-theoretic [7], known as 'unified correspondence'.

We prove the algorithmic canonicity of two classes of μ-inequalities in a constructive meta-theory of normal lattice expansions. This result simultaneously generalizes Conradie and Craig's canonicity results for μ-inequalities based on a bi-intuitionistic bi-modal language [3], and Conradie and Palmigiano's constructive canonicity for inductive inequalities [4] (restricted to normal lattice expansions). Besides the greater generality, the unification of these strands smoothes the existing proofs for the canonicity of μ-formulas and inequalities. Specifically, the two canonicity results proven in [3], namely, the tame and proper canonicity, fully generalize to the constructive setting and normal LEs. Remarkably, the rules of the algorithm ALBA used for this result have exactly the same formulation as those of [4], with no additional rule added specifically to handle the fixed point binders. Rather, fixed points are accounted for by certain restrictions on the application of the rules, concerning the order-theoretic properties of the term functions associated with the formulas to which the rules are applied.

The contributions reported on in the proposed talk pertain to unified correspondence theory [7], a line of research which applies duality-theoretic insights to Sahlqvist theory (cf. [11]), with the aim of uniformly extending the benefits of Sahlqvist theory from modal logic to a wide range of logics which include, among others, intuitionistic and distributive and general (non-distributive) lattice-based (modal) logics [8, 10], non-normal (regular) modal logics based on distributive lattices of arbitrary modal signature [23], hybrid logics [14], many valued logics [20] and bi-intuitionistic and lattice-based modal mu-calculus [3, 5].

The breadth of this work has stimulated many and varied applications. Some are closely related to the core concerns of the theory itself, such as understanding the relationship between different methodologies for obtaining canonicity results [22, 9], the phenomenon of pseudo-correspondence [12], and the investigation of the extent to which the Sahlqvist theory of classes of normal distributive lattice expansions can be reduced to the Sahlqvist theory of normal Boolean algebra expansions, by means of Gödel-type translations [13]. Other, possibly surprising applications include the dual characterizations of classes of finite lattices [15], the identification of the syntactic shape of axioms which can be translated into structural rules of a proper display calculus [18] and of internal Gentzen calculi for the logics of strict implication [21], and the epistemic interpretation of lattice-based modal logic in terms of categorization

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theory in management science [6]. These and other results (cf. [11]) form the body of a theory called unified correspondence [7], a framework within which correspondence results can be formulated and proved abstracting away from specific logical signatures, using only the order-theoretic properties of the algebraic interpretations of logical connectives.

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