

Sahlqvist via Translation

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Unified correspondence. In recent years, Sahlqvist theory has significantly broadened its scope, extending the benefits it originally imparted to modal logic to a wide range of logics which includes, among others, intuitionistic and distributive and general (non-distributive) lattice-based (modal) logics [6, 8], non-normal (regular) modal logics based on distributive lattices of arbitrary modal signature [18], hybrid logics [12], many valued logics [15] and bi-intuitionistic and lattice-based modal mu-calculus [1, 3, 2].

The breadth of this work has stimulated many and varied applications. Some are closely related to the core concerns of the theory itself, such as understanding the relationship between different methodologies for obtaining canonicity results [17, 7], and the phenomenon of pseudo-correspondence [10]. Other, possibly surprising applications include the dual characterizations of classes of finite lattices [13], the identification of the syntactic shape of axioms which can be translated into structural rules of a proper display calculus [14] and of internal Gentzen calculi for the logics of strict implication [16], and the epistemic interpretation of lattice-based modal logic in terms of categorization theory in management science [4]. These and other results (cf. [9]) form the body of a theory called unified correspondence [5], a framework within which correspondence results can be formulated and proved abstracting away from specific logical signatures, using only the order-theoretic properties of the algebraic interpretations of logical connectives.

Focus of the present talk. Notwithstanding the new insights and the connections with various areas of logic brought about by these developments, a natural question to ask is whether, just for the sake of Sahlqvist theory, it is possible to obtain Sahlqvist-type results for nonclassical logics by means of a reduction to a setting of normal modal logic via some suitable translations, such as the Gödel-Tarski. The present talk reports on the results of [11], in which this question is investigated for logics algebraically captured by normal distributive lattice expansions (DLEs). Our conclusions are that, while the most general Sahlqvist-type *correspondence* result for DLE-inequalities can indeed be obtained straightforwardly via translation, the proof of *canonicity* can be obtained as straightforward only in the special setting of normal *bi-Heyting algebra* expansions.

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