An approach to parts of d-frames and an Isbell-type density theorem *

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With the purpose of finding a Stone duality for bitopological spaces, A. Jung and A. Moshier introduced in [2] the category dFrm of *d-frames* in which objects are structures that comprise two frames, thought of as lattices of open sets, and two relations that connect both frames, as abstractions of the covering and disjointness relation. Morphisms in this category, named d-frame homomorphims, are pairs of frame homomorphisms preserving those relations. The aim of this talk is to explore an approach to the notion of parts of a space in this pointfree bitopological setting.

In the category Loc of locales, subobjects, namely sublocales, can be thought of as *generalized subspaces* and they form a far more complex and richer structure than their classical counterpart [1]. They can be represented in several different ways: frame congruences, nuclei, sublocale sets and sublocale maps (onto frame homomorphisms). The categorical interpretation of the last one provides a candidate for the bitopological case, as onto frame homomorphisms are precisely extremal epimorphisms in the category $\mathsf{Frm} = \mathsf{Loc}^{\mathrm{op}}$ of frames. Motivated by this fact, in this talk, we will present a characterization of extremal epimorphisms in the category of d-frames. They are given by certain pairs of onto frame homomorphisms and, consequently, they can be represented by pairs of sublocale sets endowed with appropriate covering and disjointness relations. However, non-trivial examples are not easily found and even though one can easily show that they form a complete lattice, this structure does not seem to be as rich as in the localic case.

Furthermore, we will define dense d-frame homomorphisms and show that, given a pair of sublocale sets containing all the regular elements of a d-frame, their associated sublocale maps form a dense extremal epimorphism in dFrm. Conversely, an extremal epimorphism in dFrm is dense if and only if its associated pair of sublocale sets contains all the regular elements. Accordingly, we will show that there is a least dense extremal epimorphism for each d-frame, obtaining in this way a Isbell-type density theorem for the category of d-frames.

References

[1] J. R. Isbell, Atomless parts of spaces, *Math. Scand.* **36**, 5–32 (1972).

[2] A. Jung, M. A. Moshier, *On the bitopological nature of Stone duality*, Technical Report CSR-06-13, School of Computer Science, The University of Birmingham (2006).

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