Many-Valued Arrow Logic with Scalar Multiplication

José David García-Cruz

National Autonomous University of México (UNAM)
México City, México
0010x0101x0000x0110@gmail.com

In order to reinforce the link between linear algebra, modal logic and many-valued logic we present an extension of Basic Arrow Logic (BAL) based on the introduction of a new modal operator of scalar multiplication and on a redefinition of the basic arrow operators. As we know since the work of Venema [11] and Marx [6], the novelty of BAL lies on the introduction of three modal operators, namely, identity ($\int$), converse ($-$), and composition ($\circ$). Along with other important extensions, like presented in [2], [8], [10], what results in this case is Many-valued Arrow Logic with Scalar multiplication (MALS).

The motivations of presenting MALS as an extension of BAL comes from two different sides. The first is due to van Benthem's infinitary operator "$\mathcal{M}, x \models \varphi^*$", presented in [3]. This modal operator is defined as a finite composition of a formula φ ("x can be C-decomposed into some finite sequence of arrows satisfying φ in $\mathcal{M}$" [3, p. 18]), we may think of this operator as a kind of scalar multiplication but, the definition do not specify nothing about how to interpret them like that. MALS make explicit the definition of scalar multiplication validating all vector spaces' axioms (some intuitions of our work are presents in [4, p. 289]). The second aspect is related with many valued logics, in specific with the evident similarities with the logic FDE [5], and its informational interpretation [9]. In this case our proposal is to define a kind of non-classical vector algebra, invalidating some intuitive properties like consistency, and showing that a non-classical vector algebra is still significant. In MALS this approach can be realized defining the operators in a more general way, following the work of Priest [7].

The plan of the talk is as follows. First we may introduce Arrow Logic with Scalar multiplication (ALS), later we may define Many-valued Arrow Logic (MAL). This two logics are also extensions of BAL and if we join the two we have MALS, this will be done in a third place. As a result, we may obtain MALS as a union of ALS and MAL, that means that, we may interpret – with a 4-valued semantics– composition as vector addition, converse as subtraction, and scalar multiplication (following van Benthem) as n-composition of φ where n ranges over the scalar magnitude.

References

[1] Aiello, M., Pratt-Hartmann, I.E., van Benthem, J. F. A. K. *Handbook of Spatial Logics*. Springer-Verlag Dordrecht, The Netherlands, (2007).

[2] Balbiani, P. & Vakarelov, D. "Dynamic extensions of arrow logic". Annals of Pure and Applied Logic 127, 1 –15, (2004).

[3] van Benthem, J. A note on dynamic arrow logic. In J. van Eijck and Visser, Eds. "Logic and Information Flow, Foundations of Computing". pp. 15 – 29. MIT Press, (1994).

[4] van Benthem, J. & Bezhanishvil, G. Modal Logics of Space. In: Aiello, M., Pratt-Hartmann, I.E., van Benthem, J. F. A. K. *Handbook of Spatial Logics*. Springer-Verlag Dordrecht, The Netherlands, (2007).

[5] Dunn, J. M. "Arrows Pointing at Arrows: Arrow Logic, Relevance Logic, and Relation Algebras". A. Baltag & S. Smets (eds.), *Johan van Benthem on Logic and Information Dynamics* .Outstanding Contributions to Logic 5, 881 - 894. Springer International Publishing Switzerland, (2014).

[6] Marx, J. M. *Algebraic Logic and Relativization*. ILLC Dissertation Series. (1995).

[7] Priest, G. "Many-valued modal logics: A simple approach". Review of Symbolic Logic 1, 2: 190 – 203, (2008).

[8] Stell, J. G. & Hofmann, D. Bi-Intuitionistic Arrow Logic. Avialable in: < http://www.comp.leeds.ac.uk/jgs/AiMLShortStellHofmann.pdf >

[9] Shramko, J. & Wansing, H. Some useful 16-valued Logics: How a Computer Network Should Think. Journal of Philosophical Logic 34: 121-153, (2005).

[10] Veloso, P. A. S., de Freitas, R. P., Viana, P., Benevides, M. & Veloso, S. R. M. "On Fork Arrow Logic and its Expressive Power". Journal of Philosophical Logic, 36: 489 - 509, (2007).

[11] Venema, Y. A crash course in arrow logic. In Arrow logic and multi-modal logic, Maarten Marx, László Pólos, and Michael Masuch (Eds.). Center for the Study of Language and Information, Stanford, CA, USA 3-34, (1997).