Situated Epistemic Updates

Igor Sedlár and Andrew Tedder

The Czech Academy of Sciences, Institute of Computer Science
Prague, The Czech Republic

Abstract. One way to model epistemic states of agents more realistically is to represent these states by sets of situations rather than possible worlds. In this paper we discuss representations of epistemic update in terms of situations. After linking epistemic update based on deleting epistemic accessibility arrows with update of situations, we discuss two specific kinds of public epistemic update; monotonic update in intuitionistic dynamic epistemic logic, and non-monotonic update in substructural dynamic epistemic logic. Our investigation is mainly conceptual, but leads to completeness results using reduction axioms, and lays the groundwork for future investigation into the concept of situated epistemic update.

1 Introduction

Agents routinely find themselves in epistemic states which are almost always (a) incomplete, often (b) inconsistent yet non-trivial, (c) inadequate to support all logically valid formulas, and (d) such that new information may undermine some of the previously accepted information even if the latter is correct.

Example 1. Alice has been told by her physician, Beth, that getting a specific type of Covid-19 vaccination is safe for her, but she has also been told by her friend Carl that it is not safe. She regards both Beth and Carl as trustworthy sources and so her epistemic state is inconsistent. Yet, it is also also incomplete (a), and so non-trivial (b), if, for instance, Alice lacks information about how the specific vaccine works. It is also not hard to imagine Alice as being ignorant about some complex logical tautologies (c). Moreover, if Alice receives the information that Carl's opinion about the vaccine is based on an untrustworthy source, Alice may well discard Carl's opinion. Suppose, however, that due to a rare and undetected condition, the vaccine under consideration actually isn't safe for Alice (d).

Epistemic logics based on classical logic, representing epistemic states of agents as sets of possible worlds, can accommodate (a), but (b)–(d) are more problematic. One way to accommodate (a)–(c) is to use relational models with *non-standard states* instead of, or in addition to, possible worlds; this approach dates back to at least [21, 17, 12]. The idea is to represent epistemic states as sets that may contain certain abstractions of possible worlds, which may be inconsistent and yet not support all information, and which may not support all validities of the underlying logic. These non-standard states can be seen as *situations* in the sense of situation semantics [3, 6]. Relational semantics of many non-classical logics use non-standard states and can be interpreted in terms of situations; e.g. see Mares' situational interpretation of the relational semantics of the relevant logic R [14]. Building on the idea of using situations to model epistemic states of agents realistically, various versions of non-classical epistemic logic have been studied, e.g. in [4, 23, 24].

When it comes to (d), some versions of Public Announcement Logic PAL based on non-classical logics have been studied recently: see PAL based on Dunn–Belnap logic [20, 22], intuitionistic logic [13, 2], and fuzzy logic [5]. All these approaches, however, consider monotonic epistemic update: what is "known" before the update remains "known" after the update. Plausibly, epistemic update logics based on relevant and other substructural logics should avoid this issue.

Punčochář [15] extends the framework of inquisitive substructural epistemic logic [16] with public-announcement-style epistemic update. His framework is a generalization of Kripke semantics accommodating also a representation of questions, focusing more on technical results than discussion of the framework. In this paper we take a step back from Punčochář's abstract inquisitive framework, and we focus on discussing situation-based Kripke-style semantics for epistemic update. In Section 2 we discuss how a situation-based representation of epistemic update derives from situation update, the process of updating situations with new information; the notion of epistemic update we will focus on is related to "arrow deletion" update, studied in the setting of classical epistemic logic in [29] and [8, 10, 11, 28]. Our discussion is abstract, not specifying the underlying notion of situation update. In Section 3, a logic of monotonic public epistemic update is outlined; the logic is a dynamic extension of positive intuitionistic modal logic with a weak negation building on a monotonic notion of situation update where information is "added to" situations. In Section 4, a logic of non-monotonic public epistemic update is outlined; the logic is a dynamic extension of a weak modal substructural logic in the vicinity of the basic relevant logic B building on a general representation of situation update by means of the ternary accessibility relation R prominent in relational models for substructural logics.

The use of situations to model epistemic update is of interest in itself, but it fits nicely into a broader project, that of modelling *situated epistemic updates*. There are two salient differences between the possible worlds approach and the situation approach, the first of which, the possibility for inconsistency and incompleteness, we've already discussed. The second is that the collection of situations is itself a conceptually richer structure than that of possible worlds. For instance, no possible worlds are parts of other possible worlds, but the same is not true of situations. More generally, a collection of situations can stand in a wider range of relations to each other than a set of possible worlds can; for instance, they can be causally or *physically* connected. In our example, Alice, when she's informed by Beth that the vaccine is safe, occupies many different situations at once, some of which properly extend others. There is the real situation comprising Alice, Beth, along with Beth's office and its contents. This situation is subsumed in a larger situation, incorporating not just Beth's office, but other parts of the building in which it's located, or other situations to which it is connected. Understood this way, situations are not just themselves finer grained than possible worlds, the fact that they stand in relations to each other, and can be part of one another, indicates that the structure of situations is also richer than those incorporating possible worlds.

There are number of interesting avenues of investigation available when we take into account this further feature of situations. For instance, we may consider the effects of updating a situation on a larger situation of which it is a part, and on the epistemic states of the agents of the larger situation. The salient concept here is what we'll call *situated update*; these are updates which take into account not just the epistemic states themselves, but also rich structure of situations. In order to give a theory of situated updates, we must first enrich the standard account of epistemic updates to accommodate situations. Here, we set aside the ambitious project of working out the theory of situated updates, and instead focus on the preliminary investigation into epistemic updates in non-classical settings, which is a required preliminary.

2 Situation update and epistemic update

A basic feature of situations is that they *support pieces of information* [6]. This allows us to impose a partial order $\sqsubseteq$ on any set of situations, with $s \sqsubseteq t$ meaning that t supports at least as much information as s. This also gives rise to a *compatibility relation* C on situations, with Cst meaning that all information supported by s is compatible with the information supported by t. It is natural to assume that C is symmetric. In what follows, we assume that pieces of information may be expressed by formulas of a language (specified later) generated by a countable set Pr of propositional variables.

Definition 1. *An* epistemic compatibility model *is* $\mathfrak{C} = (S, \sqsubseteq, C, E, V)$*, where (i)* $(S, \sqsubseteq)$ *is a non-empty partially ordered set; (ii)* C *is a symmetric binary relation on* S *such that, for all* $s, t, u \in S$*, if* Cst *and* $u \sqsubseteq s$*, then* Cut*; (iii)* E *is a function from an at most countable set* Ag *to binary relations on* S *such that, for all* $a \in Ag$ *and all* $s, t, u, v \in S$*,* $E_a st$*,* $u \sqsubseteq s$ *and* $t \sqsubseteq v$ *only if* $E_a uv$*; (iv)* V *is a function from* Pr *to subsets of* S *closed under* $\sqsubseteq$*, that is, to* $\{X \subseteq S \mid \forall s, t \in S (s \in X \ \& \ s \sqsubseteq t \implies t \in X)\}$*.*

Ag represents a set of *agents*. Informally, each situation s supports some body information *about the information available to agent* a or, equivalently, about the *epistemic state* of a. This body may be empty, contradictory and different for various s; we use the notation $a(s)$. Bodies (and pieces) of information in general may be represented by sets of situations; intuitively, a piece of information d is represented by the set of situations $S(d)$ supporting d and a body of information (or a set of pieces of information) $D = \{d_i \mid i \in I\}$ is represented by $S(D) = \bigcap_{i \in I} S(d_i)$. We think of $E_a(s) = \{t \mid E_a st\}$ as the representation of $a(s)$. It is natural to assume that $S(d)$ is closed under $\sqsubseteq$ and that $s \sqsubseteq t$ only if $a(s) \subseteq a(t)$. This justifies the "tonicity" condition stated in (iii) of the above definition. In what follows, we will not distinguish between pieces and bodies of information and their representations as $X \subseteq S$.

The *update* of individual situations by a fixed piece of information X can be abstractly represented, following dynamic logic, by a binary relation Q_X on S; $Q_X st$ says that t is a possible result of updating s with X. (Update is non-deterministic, represented by a relation rather than a function $S \to S$, since X may represent "uncertain" information, e.g. disjunctive information.) This can be lifted to sets of situations naturally: $Q_X Zt$ iff $\exists s \in Z$ such that $Q_X st$ (equivalently, $Q_X(s) = \{t \mid Q_X st\}$ and $Q_X(Z) = \bigcup_{s \in Z} Q_X(s)$).

The *epistemic update* of $a(s)$ with X is naturally represented as the update of $E_a(s)$ with X. Epistemic updates can be modelled, in the style of dynamic epistemic logic, as *transformations of models*; in our case an epistemic update with X will result in transforming each E_a into E_a^X such that $E_a^X(s) = Q_X(E_a(s))$ (or, equivalently, E_a^X is the composition of E_a with Q_X).[1]

We note that epistemic updates represented in this fashion are not necessarily *monotonic* (X may not be "known" after the update with X and some Y previously "known" may not be known after the update) nor *truthful* (an update with X may not carry information that X is in some sense "true").

3 The monotonic case: Intuitionistic updates

In this section we outline an epistemic update logic based on the general considerations of the previous section. This is a logic of *monotonic* and *public* epistemic updates, a version of intuitionistic dynamic epistemic logic with a weak negation. A logic of non-monotonic public updates will be discussed in the next section. The language $\mathcal{L}$ contains operators $\neg$ (unary), $\wedge, \vee$ (binary), K_a for all $a \in Ag$ (unary) and $[\cdot]$ (binary). The language $\mathcal{L}[\supset]$ adds a binary operator $\supset$. The set of $\mathcal{L}[\supset]$-formulas is generated by Pr using the above operators. We define $A \equiv B := (A \supset B) \wedge (B \supset A)$. Formulas $[A]B$ are read "B holds after public epistemic update with A".

Definition 2. *Fix $\mathfrak{C} = (S, \sqsubseteq, C, E, V)$. The* $\mathcal{L}[\supset]$-satisfaction relation *on $\mathfrak{C}$ is the smallest binary relation between pairs of the form $(\mathfrak{C}, s)$, where $s \in S$, and $\mathcal{L}[\supset]$-formulas such that (i) $(\mathfrak{C}, s) \models p$ iff $s \in V(p)$; (ii) satisfaction clauses for $\wedge, \vee$ and $\supset$ are as in the Kripke semantics for intuitionistic logic; (iii) the satisfaction clause for $\neg$ is*

[1] Note that in the limiting case where $Q_X st$ iff $s = t$ and $t \in X$ (suggesting that situations are "complete" and information cannot be really *added* to them), Q_X boils down to the *test* relation of Propositional Dynamic Logic and, abusing notation, epistemic update corresponds to the *relation changer* $r_X : E_a \mapsto (E_a; X?)$ in the style of dynamic logic with relation changers [27]. More liberal interpretations of Q_X link epistemic updates to various non-classical versions of dynamic logic with relation changers. The monotonic case outlined in the next section is related to intuitionistic relation changer logic studied in [9]; the non-monotonic case outlined in Section 4 is related to substructural dynamic logic with relation changers which, however, remains to be studied.

– $(\mathfrak{C}, s) \models \neg A$ *iff* $\forall t(Cst \implies (\mathfrak{C}, t) \not\models A)$;

and (iv) the rest of the clauses are:

– $(\mathfrak{C}, s) \models K_a A$ *iff* $\forall t(E_a st \implies (\mathfrak{C}, t) \models A)$
– $(\mathfrak{C}, s) \models [A]\, B$ *iff* $(\mathfrak{C}^A, s) \models B$,

where $\mathfrak{C}^A = (S, \sqsubseteq, C, E^A, V)$ *is such that* $E^A_a(s) = Q^{int}_{\mathfrak{C}(A)}(E_a(s))$, *where* $\mathfrak{C}(A) = \{s \mid (\mathfrak{C}, s) \models A\}$, $Q^{int}_X(s) = \{t \mid s \sqsubseteq t \;\&\; t \in X\}$ *and* $Q^{int}_X(Y) = \bigcup_{s \in Y} Q^{int}_X(s)$. *A formula* A *is valid in* $\mathfrak{C}$ *iff* $\mathfrak{C}(A) = S$ *of* $\mathfrak{C}$.

Hence, in the present setting, updating s with X may result in any $t \sqsupseteq s$ such that $t \in X$ and, starting with some fixed $\mathfrak{C}$, $E^A_a st$ iff there is u such that $E_a su$, $u \sqsubseteq t$ and $t \in \mathfrak{C}(A)$. (Note that, given that E_a is monotonic in the second position, this is equivalent to $E_a st \;\&\; t \in \mathfrak{C}(A)$.) The idea behind the definition of $\mathfrak{C}^A$ is that updating a situation with some piece of information means *adding* that piece of information to the situation, and that updates are *public:* in an epistemic update with X, the information that the epistemic state of each agent a is updated with X is added to *each situation in* S.

Theorem 1. *The set of formulas valid in all epistemic compatibility models is axiomatized by adding to any axiomatization of positive intuitionistic propositional logic (i) the De Morgan axiom* $(\neg A \wedge \neg B) \supset \neg(A \vee B)$ *and the Contraposition rule* $\dfrac{A \supset B}{\neg B \supset \neg A}$; *(ii) the Regularity axiom* $K_a(A \wedge B) \equiv (K_a A \wedge K_a B)$ *and the Necessitation rule* $\dfrac{A}{K_a A}$; *and (iii) reduction axioms for the update operator:*

1. $[A]\, p \equiv p$
2. $[A]\, \neg B \equiv \neg [A]\, B$
3. $[A]\, (B \star C) \equiv ([A]\, B \star [A]\, C)$, *for* $\star \in \{\wedge, \vee, \supset\}$
4. $[A]\, K_a B \equiv K_a(A \supset [A]\, B)$
5. $[A]\, [B]\, C \equiv [(A \wedge [A]\, B)]\, C$

It can be shown that epistemic updates in this setting are monotonic ($K_a B \supset [A]\, K_a B$ is valid) but it is not necessarily truthful: in assessing $[A]\, B$ in s, we do not assume that A is supported by s.[2] However, our framework can incorporate truthful updates as follows. Let $\mathcal{L}[\supset, !]$ be an extension of $\mathcal{L}[\supset]$ with a binary truthful public update operator $[\cdot\, !]$ and let $\mathcal{L}[\supset, !]$-satisfaction be defined as $\mathcal{L}[\supset]$-satisfaction with the additional clause

– $(\mathfrak{C}, s) \models [A!]\, B$ iff $\forall t \sqsupseteq s (t \in \mathfrak{C}(A) \implies (\mathfrak{C}^A, t) \models B)$

In assessing whether $[A!]\, B$ is satisfied in s, we evaluate $[A]\, B$ in each t that may result from s *by adding the information that* A *holds*. We note that $[A!]\, B \equiv (A \supset [A]\, B)$ is valid and so truthful update is expressible already in $\mathcal{L}[\supset]$.

[2] In fact, modifying the satisfaction clause for $[A]\, B$ to $(\mathfrak{C}, s) \models [A]\, B$ iff $(\mathfrak{C}, s) \not\models A$ or $(\mathfrak{C}^A, s) \models B$ would make $[A]\, B$ non-persistent, that is, we could have $s \sqsubseteq t$ such that $[A]\, B$ is satisfied in s but not in t.

4 The non-monotonic case: Substructural updates

In this section we outline a logic of *non-monotonic* public epistemic updates, which will be a version of substructural dynamic epistemic logic. Similarly as the logic of Sect. 3, we will follow the general "template" discussed in Sect. 2, but this time we will rely on a more general notion of situation update. The language $\mathcal{L}[\rightarrow, \circ]$ adds to $\mathcal{L}$ two unary operators $\rightarrow$ and $\circ$. We define $A \leftrightarrow B := (A \rightarrow B) \wedge (B \rightarrow A)$.

Definition 3. *An* epistemic Routley–Meyer model *is* $\mathfrak{M} = (S, \sqsubseteq, L, C, R, E, V)$ *where* $(S, \sqsubseteq, C, E, V)$ *is an epistemic compatibility model,* $L \subseteq S$ *is closed under* $\sqsubseteq$ *and* R *is a ternary relation on* S *such that:*

- $s \sqsubseteq t$ *iff* $\exists x(x \in L \ \& \ Rxst)$
- $s' \sqsubseteq s$, $t' \sqsubseteq t$, $u \sqsubseteq u'$ *and* $Rstu$ *only if* $Rs't'u'$

$\mathfrak{M}$ *is* fully associative *iff* $Rstuv \iff Rs(tu)v$*, where* $Rstuv := \exists w(Rstw \ \& \ Rwuv)$ *and* $Rs(tu)v := \exists w(Rswv \ \& \ Rtuw)$*.*

Definition 4. *A* $\mathcal{L}[\rightarrow, \circ]$-interpretation *on* $\mathfrak{M}$ *uses the same clauses as* $\mathcal{L}[\supset]$*-interpretation for* $p, \neg, \wedge, \vee$ *and* K_a*; the rest are as follows:*

- $(\mathfrak{M}, s) \models A \rightarrow B$ *iff* $\forall t, u(Rstu \ \& \ (\mathfrak{M}, t) \models A \implies (\mathfrak{M}, u) \models B)$
- $(\mathfrak{M}, s) \models A \circ B$ *iff* $\exists t, u(Rtus \ \& \ (\mathfrak{M}, t) \models A \ \& \ (\mathfrak{M}, u) \models B)$
- $(\mathfrak{M}, s) \models [A] B$ *iff* $(\mathfrak{M}^A, s) \models B$

where, for each $\mathfrak{M} = (S, \sqsubseteq, L, C, R, E, V)$*,* $\mathfrak{M}^A = (S, \sqsubseteq, L, C, R, E^A, V)$ *such that* $E^A_a(s) = Q^{sub}_{\mathfrak{M}(A)}(E_a(s))$*, where* $\mathfrak{M}(A) = \{s \mid (\mathfrak{M}, s) \models A\}$*,*

$$Q^{sub}_X(s) = \{s\} \otimes X = \{u \mid \exists t(Rstu \ \& \ t \in X)\}$$

and $Q^{sub}_X(Y) = \bigcup_{s \in Y} Q^{sub}_X(s)$*. A formula* A *is* valid in $\mathfrak{M}$ *iff* $L \subseteq \mathfrak{M}(A)$*.*

Routley–Meyer models are standard semantic structures in the relational semantics for substructural logics [18]; some classes of these models have been interpreted in terms of situations [14]. Substructural implication $\rightarrow$ and the relation R used in its satisfaction clause are sometimes read in terms of update; e.g. Dunn and Restall point out that "perhaps the best reading [of $Rstu$] is to say that the combination of the pieces of information s and t (not necessarily the union) is a piece of information in u" [7, p. 67]. Restall adds that "a body of information warrants $A \rightarrow B$ if and only if whenever you *update* that information with new information which warrants A, the resulting (perhaps new) body of information warrants B" [19, p. 362] (notation adjusted). The point has been more deeply investigated by Aucher, for instance in [1]. Hence, it is natural to use R in a general representation of situation update $\{s\} \otimes X = \{u \mid \exists t(Rstu \ \& \ t \in X)\}$. Without assuming further properties of R substructural updates are not monotonic (i.e. $K_a A \rightarrow [B] K_a A$ is invalid), thus accommodating feature (d) of Example 1.

Theorem 2. *The set of formulas valid in all epistemic Routley–Meyer models is axiomatized by adding to any axiomatization of positive basic relevant logic* B^+ *(i) the De Morgan axiom* $(\neg A \wedge \neg B) \supset \neg(A \vee B)$ *and the Contraposition rule* $\dfrac{A \supset B}{\neg B \supset \neg A}$*; (ii) the Regularity axiom* $K_a(A \wedge B) \leftrightarrow (K_a A \wedge K_a B)$ *and the Monotonicity rule* $\dfrac{A \to B}{K_a A \to K_a B}$*; (iii) reduction axioms for* $[\,\cdot\,]$*:*

1. $[A]\,p \leftrightarrow p$
2. $[A]\,\neg B \leftrightarrow \neg [A]\,B$
3. $[A]\,(B \star C) \leftrightarrow ([A]\,B \star [A]\,C)$, *for* $\star \in \{\wedge, \vee, \to, \circ\}$
4. $[A]\,K_a B \leftrightarrow K_a(A \to [A]\,B)$

and the Update congruence rule $\dfrac{A \leftrightarrow A' \quad B \leftrightarrow B'}{[A]\,B \to [A']\,B'}$.

The set of formulas valid in all fully-associative *models is axiomatized by the system obtained from this by replacing the Update congruence rule by:*

5. $[A][B]\,C \leftrightarrow [(A \circ [A]\,B)]\,C$

A substructural version of truthful announcements can be incorporated as before. Let $\mathcal{L}[\to, \circ, !]$ be an extension of $\mathcal{L}[\to, \circ]$ with $[\cdot\,!]$. The definition of $\mathcal{L}[\to, \circ, !]$-interpretation adds the clause:

– $(\mathfrak{M}, s) \models [A!]\,B$ iff $\forall t, u(Rstu \;\&\; t \in \mathfrak{M}(A) \implies (\mathfrak{M}^A, u) \models B)$.

As before, in assessing the effects of truthful announcements of A in s, we update s itself with A; this time, however, a different notion of situation update is used. Similarly as before, $[A!]\,B \leftrightarrow (A \to [A]\,B)$ is valid.

5 Conclusion

This paper has laid the groundwork for a theory of epistemic updates employing situations rather than possible worlds. While interesting as a topic in its own right, for our purposes this serves mainly as a necessary preliminary for the investigation of properly *situated* epistemic updates, taking into the account the richer modelling apparatus at which we have here only gestured. Beyond this, two extensions of the present work are natural. Firstly, one may consider an extension of the epistemic update logics studied here, especially the relevant ones, with *common knowledge*. Notoriously, completeness results for such logics cannot be established using only reduction axioms. We conjecture that a combination of reduction axioms with a "partial filtration" proof strategy (developed in [25, 26]) will work. Second, the "relevant" rendering of situation update Q_A connects relevant epistemic update with the notion of "arrow test" considered in [25, 26]. This motivates a generalization to a relevant dynamic logic of relation changers.

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