Fuzzy Semantics for Graded Predicates

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Some predicates apply to the world in degrees. For instance, *warm*, *bent* and *acute* all appear to be graded in this sense, as can be seen from the fact that they can figure in statements such as *Last Winter was the warmest ever recorded*, *The rod is slightly bent*, *A 30° angle is more acute than a 60° one*. Graded predicates have been the subject matter of important debates in three different, although overlapping, disciplines: philosophy, linguistics and mathematical logic. Unsurprisingly, each of these takes a different approach to the matter. Despite the previous efforts to build bridges between the different communities, we believe there is room for more. In particular, the link between mathematical logic and linguistic semantics should be further explored. In this talk, we aim to do that by sketching a way in which linguistic semantics could benefit from recent developments in mathematical fuzzy logic. Thus we adopt the aims of the linguists (and consider the wealth of data they have gathered) while capitalising on the tools of the fuzzy logician.

The very question of how graded predicates should be categorised is an interesting one. To give a sense of the complexity of the phenomenon at hand, let us sketch what we take to be the best taxonomy of graded predicates.[1] Firstly, we distinguish predicates whose applicability can be measured from those (if there are any) whose applicability is not measurable. Uncontroversially, predicates denoting physical qualities fall among the former. Controversially, perhaps moral and aesthetic adjectives fall among the latter. Amongst the measurable ones, we distinguish uni- from multi-dimensional ones. The former are those whose degree of applicability varies along a single scale; e.g. *tall* (scale: linear extent). By contrast, the latter have various underlying scales; e.g. *intelligent* (scales: memory, arithmetical skill, etc). Among both uni- and multi-dimensional predicates we draw distinctions according to the features of their underlying scales. In particular, we distinguish linear from non-linear scales. Although this is controversial, an example of a uni-dimensional non-linear predicate could be *painful* – not all painful events are comparable and yet there seems to be a single scale of pain. Turning to linear predicates, we distinguish between vague and precise predicates. Vague predicates are characterised (among others) by having blurry boundaries. We have already seen some: *tall* and *warm*. Precise predicates draw sharp divides between their extensions and their anti-extensions but still rely on a scale of degrees for their applicability. Among these we find, as a limiting case, bivalent predicates (e.g. *even number*), but also non-bivalent ones. In turn, the latter are divided into at least three sorts: predicates which demand to reach a maximum amount of a certain quality for their applicability (e.g. *full*), those which instead demand to surpass a minimum amount (e.g. *dirty*) and those whose turning point is neither the minimum nor the maximum of the scale (e.g. *acute angle*).

After decades of being ruled out due to objections such as that of artificial precision, the fuzzy approach to vagueness was reexamined and vindicated in the form of fuzzy plurivaluationism (Smith, 2008). This approach takes, instead of a single fuzzy model, a set of various fuzzy models for the semantics of a vague predicate (thereby overcoming the artificial precision problem). While this appears to have revived the interest in fuzzy logic

[1] Our categorisation stems, in part, from Paoli (1999) and Kennedy and McNally (2002).

as a tool for vagueness to some extent, its potential in formal semantics remains virtually unexplored. In this talk, we take Smith's reformulation of the fuzzy theory of vagueness (and his treatment of the sorites paradox) as our starting point and make adjustments to it in order to turn it into a more complete semantic theory; one which extends to all the sorts of graded adjectives enumerated above and the wide range of linguistic constructions where they appear.

Interestingly, what nowadays can be taken as the most widely accepted linguistic semantics for graded predicates makes use of scales of degrees.[2] We call it 'degree-based semantics'. This fact suggests a connection with the fuzzy approach, but the former is ultimately classical: the (classical) truth-value of a statement involving a graded predicate is evaluated on a scale of degrees and is based on a contextually given degree which acts as a standard of comparison (e.g. being tall is having the quality of tallness to a degree higher than or equal to the standard of comparison). We see this as a loss. Before presenting our own proposal we will point out some weaknesses of degree-based semantics in order to motivate our alternative proposal. Among others, we will argue, following Smith (2008), that their contextualist treatment of the sorites paradox is unsatisfactory.

Our own proposal is to fuzzify the degree-based approach, in a certain sense. In order to do this we will take (reconstructions of) their degree scales as being truth scales. That is, in our account, truth-degrees are not computed on the basis of these other degrees, but, rather, *are* those degrees. This alone simplifies substantially the degree-based analysis, for it allows to drop the notion of standard of comparison. Under our proposal, adjectives are predicates, that is, functions from the domain of individuals onto a certain structure of truth-values. Degree-based theories, by contrast, take adjectives to denote measure functions (functions from individuals to scales of degrees), which are, later on, used in the analysis of predicates. In order to achieve this, they need to appeal to a null degree morpheme that, when attached to a measure function, turns it into a predicate. The fact that our account allows for this simplification of the analysis of basic predicates and that it provides a philosophically more satisfactory treatment of the sorites paradox are important reasons to explore its potential. And this is what we do in the rest of the talk.

We will take the truth scales associated with each kind of adjective to be subalgebras of a UL-chain $\mathbf{B} = \langle B, \wedge, \vee, \&, \rightarrow, \overline{1}, \overline{0}, \top, \bot \rangle$, where

(1) $\langle B, \wedge, \vee, \bot, \top \rangle$ is a bounded lattice.
(2) $\langle B, \&, \overline{1} \rangle$ is a commutative monoid.
(3) $z \leq x \rightarrow y$ iff $x \& z \leq y$. (residuation)
(4) $((x \rightarrow y) \wedge \overline{1}) \vee ((y \rightarrow x) \wedge \overline{1}) = \overline{1}$. (prelinearity)

Moreover, $\leq$ is a total order. Note that $\top$ and $\bot$ denote the maximum and the minimum of the order, respectively, and $\overline{1}$ and $\overline{0}$ serve to define the filter of the algebra (as $F = \{b \in B : b \geq \overline{1}$) and to define negation (as $\phi \rightarrow \overline{0}$), respectively.

For instance, we take vague predicates as denoting functions $f : D \rightarrow B$, where the range of f is a subset of the whole B. By contrast, precise non-bivalent predicates denote functions $f : D \rightarrow B$, where the range of f is more restricted: it is a subset of $\{x : x \leq 0 \text{ or } 1 \leq x\}$. And, as one would expect, bivalent predicates denote functions $f : D \rightarrow B$, where the range of f is {0,1}. Note that these constructions allow us to identify vague predicates as the only ones that satisfy the property of closeness,[3] for they are the only ones for which a small change in aspects relevant to the property in question brings about an equally small change in respect of truth.

[2] It has been advocated, among others, in Kennedy (2007) and Kennedy and McNally (2002).
[3] Which has been argued to be characteristic of vagueness by Smith (2008).

Armed with these structures, we can analyse any adjective *F* as denoting a function **F** from the domain of individuals to a subalgebra of B: $[\![_{A}F]\!] = \lambda x.\mathbf{F}(x)$. Thus, in this framework, the positive unmarked form (e.g. *be warm*) is simply analysed as the adjective itself. No other account of graded adjectives we are aware of provides such a simple treatment of the positive position.

Also very naturally, we analyse the comparative as a binary relation between degrees: $[\![_{DM}\text{-}er/more]\!] = \lambda\mathbf{X}\lambda\mathbf{Y}\lambda x\lambda y.\mathbf{X}(x) > \mathbf{Y}(y)$.[4] As one would expect, the superlative will be analysed as a function of the comparative by making use of an implicit comparison class: $[\![_{DM}\text{-}st/most]\!] = \lambda\mathbf{X}\lambda x.\forall y \in C(\mathbf{X}(x) > \mathbf{X}(y))$, where C is a contextually given class of relevant objects with respect to which the argument of the superlative is being asserted to be the most *X*. Finally, degree modifiers such as *very/slightly/quite/extremely* are analysed as mapping scales onto other scales. For instance, $[\![_{DM}very]\!] = \lambda\mathbf{X}\lambda x.\mathbf{very}(\mathbf{X})(x) = \lambda\mathbf{X}\lambda x.((\mathbf{X})(x))^k$, for some suitable k.[5]

After looking at degree morphology, we will turn to the analysis of sentences involving logical connectives. At this point we will need to make a choice of logic. First, note that Gödel logic and Product logic will not do here, due to their treatment of negation – i.e. according to them, *tall* would be vague, but not *not tall*. The obvious alternative is Łukasiewicz logic. This logic has the advantage that it gives us a correct analysis of negated predicates and that, on top of that, it provides quite satisfactory answers to the objections to truth-functionality in the presence of borderline cases.

We will close our talk by responding to potential objections that could be raised against our account and by pointing out a couple of limitations: the problem of comparative trichotomy (i.e. a good theory should predict that a sentence like *The wall is wider than the water is warm* is semantically anomalous) and the problem that our account does not cover adjectives associated with scales that are non-linear. Interestingly, both of these problems point to the same solution: further liberalize the algebra of truth values to one that is non-linear. This would allow for two scales to be such that their degrees are incomparable and it would open the door to having a scale within which there are elements that are incomparable. If such models could be described in a somewhat constructive manner, we believe they could provide a very powerful tool to analyse the huge variety of linguistic phenomena mentioned in this outline.

References

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[4] We will also explore an alternative analysis of the comparative which tracks the difference between degrees. This would allow us to analyse modified comparatives like *slightly/much taller* and differential comparatives like *3 cm taller* as functions of the basic comparative construction.

[5] For the analysis of vague hedges, we will mostly follow Lakoff (1973).