

Notes about point-free topology

Abstract

Apologies: There will not be much about logic.

Motivation. (A) What really happened. Logic of first and higher orders, etc. Tendency of (classical) topology to sets of sets, or sets of sets of sets. Make the order lower.

(B) What should have happened (fairy tale). Topology is a generalized geometry. Geometry was first synthetic, only much later analytic. Topology is just analytic, why not think about a synthetic one?

Short history. Wallman, M.H. Stone (late thirties, forties)

Late fifties, sixties: the real start (many names). Space determined by the lattice of open sets (Thron, Bruns). How it develops now.

Definition, synthetic topology. Space as the structure of realistic spots (places). Definition of a frame. Frame homomorphisms and continuity.

Questions one has to ask, and answers. (1) The concept is (basically) more general. Is the generalization desirable at all? (2) Is the theory in some sense (and in results) more satisfactory? (3) Doesn't the algebraic technique obscure the geometric contents? (4) Do we not lose too much information? Answers will be given, with examples.

In particular to (2): constructibility. It turned out that some heavily non-constructive facts now do not need AC. Example, explanation. Approximation.

Just to have some logic. Vickers: frames as a background of logic of finite decisions.

Jung, Moshier and others: d-frames as a background of knowledge-bases for expert systems.