

What is the height of Gentzen's reduction trees?

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Abstract

Gentzen's consistency proof of 1935 uses cut elimination in an infinitary calculus to obtain the consistency of Peano arithmetic. The cut elimination theorem is called the *Hilfssatz* and it is interesting because of two reasons:

- (1) The cut elimination strategy applied there eliminates always an uppermost cut regardless of the complexity. This is in contrast to the commonly used cut elimination strategy, called Tait's strategy, that eliminates one of the most complex cuts.
- (2) The proof of the *Hilfssatz* makes implicit use of transfinite induction up to the height of cut free infinitary derivations that have been already constructed.

If Gentzen had applied Tait's strategy in the *Hilfssatz*, he would have obtained transfinite induction up to ϵ_0 . Based on the analysis of Gentzen's cut elimination strategy, we want to explain that Gentzen's original proof might require transfinite induction up to some ordinal that is bigger than ϵ_0 . Another interesting question is whether and how both cut elimination strategies differ in finite calculi.