Abstracts: New Directions in Relevant Logic

- Ethan Brauer

  Title: The Comparative Study of Relevance in Proof Theory

  Abstract: In the context of an argument, we can say some component is irrelevant when it contributes nothing to the validity of the argument. This informal idea can be formally explicated in multiple ways, depending on which components we attend to and how strongly we understand the notion of contributing to. I will explain three explications of increasing strength, called relevant validity, perfect validity, and gaunt validity. These notions are all defined against a background notion of validity (such as classical, constructive, etc.). Thus, their primary interest lies in studying the sorts of deductions that do or not exhibit those different relevance properties. As one example of this proof-theoretic study of relevance, I outline a method for transforming proofs into proofs of gauntly valid sequents, and discuss the range and limitations of the method.

- Fabio De Martin Polo

  Title: On the Proof Theory for Relevant Logics

  Article: The aim of this presentation is to perform a proof theoretic investigation of a wide number of relevant logics by employing the well-established methodology of labelled sequent calculi to build our intended systems. At the semantic level, we will characterise relevant logics by employing reduced Routley-Meyer models, namely, relational structures with a ternary relation between worlds along with a unique distinct element considered as the real (or actual) world. This talk realizes the idea of building a variety of modular labelled calculi by reflecting, at the syntactic level, semantic informations taken from reduced Routley-Meyer models. Central results include proofs of soundness and completeness, as well as a proof of cut-admissibility.

- Nicholas Ferenz

Title: Quantified Modal Relevant Logics II: Neighbourhoods

Abstract: Building off the Mares-Goldblatt interpretation of the quantifiers introduced in [7, 6], I have previously constructed a semantics for quantified modal relevant logics based on fully *relational frames* [2]. More recently, and with Andrew Tedder ([8, 3]), I have been working to generalize these results to *fully neighbourhood frames*. That is, frames where the ternary relation and each binary relation have been 'neighbourhoodized'. The Mares-Goldblatt approach to quantification offers a simple and elegant semantics, solves problems of incompleteness for constant domain semantics (e.g., see [4]), and plays well with modalities. In this paper, we extend previous work by investigating general neighbourhood semantics for quantified modal relevant logics. The main results are (i) general modular proofs of soundness and completeness, (ii) frame-wise equivalence results between relational frames and augmented neighbhourhood frames, and (iii) independence proofs between augmentation and Barcan formulas. Moreover, I will discuss some recent adjacent results/papers, and some philosophical interpretations and upshots. The range of logics for which we prove modular soundness and completeness is large, and includes any logic obtainable from the basic system **F** of [8] (called "Min" in [5]) by any of the quantifier or modal axioms/rules mentioned in [2], and a range of others. We use the tools of gaggle theory, see [1], to achieve our frame-equivalence results.

References

[1] Katalin Bimbó and J. Michael Dunn. *Generalized Galois Logics: Relational Semantics for Nonclassical Logical Calculi.* CSLI (2008).

[2] Nicholas Ferenz. "Quantified Modal Relevant Logics". *Review of Symbolic Logic* (Forthcoming).

[3] Nicholas Ferenz and Andrew Tedder. "Neighbourhood Semantics for Modal Relevant Logics". *Journal of Philosophical Logic* (Forthcoming).

[4] Kit Fine. "Incompleteness for Quantified Relevance Logics". *Directions in Relevant Logic*, ed. J. Norman and R. Sylvan, pp. 205–225. Kluwer (1989).

[5] Lou Goble. "Neighborhoods for Entailment". *Journal of Philosophical Logic* 32:483–529 (2003).

[6] Robert Goldblatt and Edwin D. Mares. "A General Semantics for Quantified Modal Logic". *Advances in Modal Logic* v.6 (2006).

[7] Edwin D. Mares and Robert Goldblatt. "An Alternative Semantics for Quantified Relevant Logic". *Journal of Symbolic Logic* 71(1):163–187 (2006).

[8] Andrew Tedder and Nicholas Ferenz. "Neighbourhood Semantics for First-Order Relevant Logics". *Journal of Philosophical Logic* 51:457–484 (2022).

- Thomas Macaulay Ferguson

  Title: Relevance and Containment: Half a Century Later

  Abstract: Containment logics in the vicinity of Parry's analytic implication have always been acknowledged as a sort of sibling to relevant logics proper. A coarse syntactic case can be made for this acknowledgement as Parry's Proscriptive Principle entails the Variable Sharing Property. In the past half century, a great deal of model-theoretic understanding of Parry's logics has been won due to investigations into the semantics of topic or subject-matter. In this talk, I wish to bring this work to bear on understanding relevance as overlap of topic, emphasizing the consequences for several syntactic characterizations of relevance.

- Shay Logan

  Title: Relevant Restricted Universals from a Quantificational Perspective

  Abstract: The purpose of this talk is to sketch a novel semantics for relevant restricted universals. I begin with a very brief sketch of a useful philosophical interpretation of Kit Fine's stratified semantics for first-order relevant logics. The picture suggests a natural avenue to try, which I then explore in the remainder of the talk. Remarkably, almost everything works out. I end by sketching what doesn't work and what remains to be done.

- Franci Mangraviti

  Title: Consistent Theories in Inconsistent Logics

  Abstract: Logics with theorems which contradict other theorems (so-called "inconsistent logics") have come up for investigation recently. Notable examples include several connexive logics [Mor84] [NW2X], Abelian logic [MS89], and second-order **LP** [HP18]. On a classical Tarskian notion of theory closure, the inconsistency necessarily spreads from the logic to every theory built on it; under a relevant notion of theory closure, however, it is not a given that a theory contains all the axioms of its underlying logic, and so it is not a given that a theory inherits the inconsistency of said logic.

  In this talk, I am going to present some particular and general results on the existence of consistent theories for a variety of inconsistent logics, the main examples being propositional relevant connexive logics and quantified Abelian logic. These initial results suggest that the interaction between the inconsistency of a logic and the consistency status of its theories might prove interesting from a technical perspective, and that there might be room for a theorist to adopt inconsistent logics without thereby being committed to any inconsistency.

  References

  [HP18] Allen P Hazen and Francis Jeffry Pelletier. Second-order logic of paradox. *Notre Dame Journal of Formal Logic*, 59(4):547–558, 2018.

  [Mor84] Chris Mortensen. Aristotle's thesis in consistent and inconsistent logics. *Studia Logica* 43(1):107–116, 1984.

  [MS89] Robert K Meyer and John K Slaney. Abelian logic (from A to Z). In *Paraconsistent Logic: Essays on the inconsistent*, pages 245–288. Philosophia Verlag, 1989.

  [NW2x] Satoru Niki and Heinrich Wansing. On the provable contradictions of the connexive logics c and c3. *preprint*, 202x.

- Edwin Mares

  Title: E and R: Combining their Semantics

  Abstract: In my forthcoming book, The Logic of Entailment and its History, I motivate a version of Fine's semantics for the logic E. A referee asked the reasonable question how is this view to be combined with my defence of the Routley-Meyer semantics for the logic R. This paper answers the referee's question, showing how a single model can be constructed to incorporate both of these semantical theories and suggests an alternative way of understanding necessity in relevant logic.

- Ondrej Majer

  Title: TBA

  Abstract: TBA

- Tommaso Moraschini

  Title: The Relevance Principle Meets Structural Completeness

  Abstract: A classical result of Łoś and Suszko states that the algebraic counterpart of a general formulation of the relevance principle is the joint embeddability property. We investigate the relation of the latter with Wroński's notion of passive structural completeness, i.e., the demand that every rule whose antecedent cannot be unified is derivable. The results of this talk have been collected in the paper T. Moraschini, J.G. Raftery, and J.J. Wannenburg. Singly generated quasivarieties and residuated structures. Mathematical Logic Quarterly, 66(2):150-172, 2020.

- Tore Fjetland Øgaard

  Title: The Significance of the Use Criterion

  Abstract: Anderson and Belnap set forth two formal relevance properties—the variable sharing property and the Entailment theorem, but whereas the former has been unanimously adopted by relevant logicians, the latter was regarded as insignificant by the Routley-tradition. It's motivating idea, however—the *use-account* of relevance—continues to be appealed to by some relevant authors. I think the Routley-tradition was too quick to dismiss the use-idea of relevance, and so in my talk I will go through some of the history of the idea as well as the formal property motivated by it to try to shed light on its true importance for the philosophy of relevant logics.

- Takuro Onishi

  Title: Collection-collection Frames

  Abstract: In this talk, I consider an extension of Restall and Standefer's collection frames. A collection frame is a set of points equipped with a relation that relates a collection (tree, list, multiset, set, etc.) of points and a point, satisfying the conditions of reflexivity and compositionality. I will extend their multiset frame for RW+ and R+ to *multiset-multiset* frame, namely a frame with a relation between multisets, like a multiple conclusion sequent. I will show that a multiset-multiset frame with reflexivity and (a suitably modified version of) compositionality can be constructed from a *GS frame* for RW and R I presented earlier, in which the star function is definable thanks to the interaction between two dual ternary relations. This implies that RW and R are (sound and) complete with respect to the corresponding class of multiset-multiset frames.

- Vít Punčochář.

  Title: Relevant Epistemic Logic

  Abstract: In this talk I will focus on the problem of modelling various epistemic operators, like public announcement, common knowledge and a question generating operator, in the context of relevant propositional logics. I will address this problem from both syntactic and semantic perspectives and I will discuss technical as well as philosophical issues related to this problem. The talk builds on recent collaboration with Igor Sedlár and Andrew Tedder.

- Greg Restall

  Title: TBA

  Abstract: TBA

- Gemma Robles and José M. Méndez

  Title: The Lattice of all 4-Valued C-Extending Implicative Expansions of Belnap-dunn Logic Containing Routley and Meyer's Basic Logic B^d

  Abstract: The well-known logic FDE, introduced by Belnap and Dunn, is defined with $\wedge, \vee$ and $\sim$ as the sole primitive connectives. Then, the question of expanding FDE with an implication connective of course of great interest. In this sense, some implicative expansions of FDE have been proposed in the literature, among which Brady's logic $BN4$ seems to be the preferred option of relevant logicians. The aim of this paper is (1) to find the class of all 4-valued C-extending implicative expansions of FDE verifying the axioms and rules of Routley and Meyer's basic logic B and its disjunctive extension B^d (this class is labeled $ME4B^d$); (2) to establish the lattice of $ME4B^d$ by relying upon the functional relations the members in this class maintain to each other. (An implicative function is C-extending if it coincides with the function for the classical conditional when restricted to the "classical values" T and F; (3) to show how to use two-valued Belnap-Dunn semantics into order to give a Hilbert-style proof system for each one of the logics $ME4B^d$ determines. We remark that Boolean negation is definable in the strongest element in $ME4B^d$ and so it can be axiomatized as an expansion of classical propositional logic.

- John Slaney

  Title: Relevant Number Theory: Beyond R#

  Abstract: The arithmetic R# is obtained by postulating the Peano axioms on the basis of the relevant logic R. R# is a remarkable arithmetic, not least in that it has finite models. In this paper we examine the options for extending R# from natural numbers to rational numbers, as this is the essential next step towards providing a relevant basis for mathematics and for applications. Relevant rational number theory is problematic in that the most obvious approaches lead to non-conservative extensions

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of R#. We consider three ways in which relevant theories of rational arithmetic can be formulated, and note in particular how these fare in the finite models of R#.

- Neil Tennant

  Title: Core Proofs as Objects of Search: Preserving Relevance and Epistemic Gains

  Abstract: We explain how the model-invariant rules of inference of Core Logic (in natural deduction: introduction and elimination rules; in sequent calculus: Right and Left rules) arise naturally from the model-relative rules of evaluation (verification and falsification) by simple 'morphing'. We then explain how two important features of the rules for core proof arise from constraints on 'bottom-up' proof search. Relevance of premises to conclusions is always preserved. So too are any epistemic gains made when pursuing solutions to the deductive sub-problems posed in the course of search.

- Pilar Terrés Villalonga.

  Title: Naturalizing, or the Connection between Two Routes to Relevance

  Abstract: In this talk I present and connect two main routes to relevance that we find in the literature: the core route and the ambiguous route. The former establishes relevance at the level of consequence, departing from a non-relevant logic and discriminating those inferences which comply with certain relevant criteria; the later route constructs relevance at the level of logical connectives, rejecting weakening, and building a logical vocabulary which internalizes this rejection, making certain logical connectives split into intensional and extensional counterparts. I argue that these two routes are intimately connected: the set of valid sequents that result from translating each intensional/extensional connective in relevant logic obtained from the second route into the same connective (a process that we refer as the naturalization of a logic) coincides with the relevant core of the first route, which shows how the two alternatives impose the same norms to how we should reason with logical connectives in natural language.

- Peter Verdée

  Title: Grounding as a Guide to Relevance

  Abstract: In this talk I will give an overview of our project of characterizing relevance in terms of a theory of logical grounding. We will show that interesting (new and old) relevant entailment relations can be obtained from existing logical languages, if one characterizes the meaning of those languages by means of an underlying theory of logical grounding. The notion of relevance thus obtained is not based on specificities of the meaning of logical languages, but tries to get at the concept of relevant entailment itself. Central in our concept of relevance is the idea of joint-contribution: for a valid argument to be relevantly valid all premises and conclusions must jointly contribute to the validity, by exchanging their potential grounds. We will work this out formally, show what this gives in concrete cases and discuss the relation between our approach

and existing approaches to relevance (with a focus on the Anderson & Belnap tradition and on Neil Tennants approach).

- Jamie Wannenburg

  Title: TBA

  Abstract: TBA