

# Surrogate data in detecting nonlinearity and phase synchronization

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## Abstract

A hypothesis testing approach utilizing the technique of surrogate data is used for detecting nonlinearity and phase synchronization in bivariate time series. Instantaneous phases are obtained by means of discrete Hilbert transform. Information-theoretic functionals – redundancies are used as the test statistics. Described methods are illustrated in detecting certain nonlinearities and synchronization in cardio-respiratory interactions in the case of a newborn piglet during quiet sleep.

Keywords: surrogate data, redundancy, mutual information, nonlinearity, phase synchronization, cardio-respiratory interaction

## 1 Introduction

Methods for detecting nonlinearity in univariate and multivariate time series, and for identification of phase synchronization in bivariate time series are presented, which both utilize the technique of surrogate data. The detection of nonlinearity is based on rejecting a null hypothesis of a linear stochastic process with the same spectra and cross-spectra as the studied series, while the phase synchronization is identified especially by rejecting the hypothesis of linear stochastic processes asynchronously oscillating on the same frequencies (spectra) as the series under study. Instantaneous phases are obtained by means of the discrete Hilbert transform. Information-theoretic functionals – redundancies are used as the test statistics. The methods are illustrated in analysis of bivariate animal cardio-respiratory data reflecting interactions between respiratory movements and heart rate fluctuations, which are considered as an important aspect of the functional organization in the autonomous nervous system. By means of the methods presented here certain nonlinearities and synchronization of cardio-respiratory interactions were found in a newborn piglet during quiet sleep.

## 2 Material

In a newborn piglet ECG was measured by standard electrodes and sampled at 2048 Hz in order to get sufficient resolution of the instantaneous heart rate fluctuations. The QRS-complexes were detected by the steepest ascent criterion and the instantaneous heart rate series was calculated from the RR-interval series. This heart beat related series was transformed into an equidistant time series by means of interpolation corresponding to the pacemaker process in the heart. The respiratory movements were measured by using impedance respirography and sampled at 128 Hz. Both signals (respiratory movement and heart rate fluctuation) were step by step low pass filtered down to 4 Hz (phase correct FIR-filter), synchronously resampled at 8 Hz, and then used for the analysis. In order to investigate a well defined behavioral state

the experimental data were preclassified into different sleep states and the cardio-respiratory data of one certain interval of quiet sleep was selected for the analysis.

### 3 Surrogate Data

The surrogate data method, related to the technique of bootstrap [1, 2], has been methodologically introduced into nonlinear dynamics by Theiler et al. [3] as a method for testing nonlinearity. The basic idea in the surrogate-data based nonlinearity test is to compute a *nonlinear* statistic for data under study and for an ensemble of realizations of a linear stochastic process, which mimics “linear properties” of the studied data (“null hypothesis”). If the computed nonlinear statistic for the original data is significantly different from the values obtained for the surrogate set, one can infer that the data were not generated by a linear process; otherwise the null hypothesis, that a linear model fully explains the data is accepted.

In general, surrogate data are artificially generated data, which mimic (a part of) statistical properties of the data under study, but not the property which is tested for. If *any* temporal dependence is under question, a test can use the null hypothesis of an independent identically distributed (IID) process (strictly white noise) and so called *scrambled* surrogates are used. In the case of testing for nonlinearity, the surrogate data should have the same spectrum and, consequently, the same autocorrelation function (“linear properties”) as the original data under study.

Studying bivariate phenomena such as cardio-respiratory synchronization we define following types of the surrogate data:

1. *IID1 surrogates* are realizations of mutually independent IID stochastic processes (white noises) which preserve sample means, variances and histograms of the series under study. The IID1 surrogates are constructed by “scrambling” the original series, i.e., the elements of the original series are randomly permuted in temporal order, in each realization different random permutations are used for the two components of the bivariate series. This randomization destroys any temporal structure, if present in the original series. The IID1 surrogates represent the null hypothesis of independent strictly white noises, i.e., nor synchronization neither oscillations are considered.

2. *IID2 surrogates* are realizations of IID stochastic processes which count for possible cross-dependence between the two components of the bivariate series. In each realization, the same random permutation is used for both components of the bivariate series. The IID2 surrogates present the null hypothesis of mutually dependent white noises, i.e., the two series are “synchronized” in a sense of mutual dependence given, e.g., by crosscorrelations.

For better understanding, consider a “toy” example – two three-sample series  $\{a, b, c\}$  and  $\{A, B, C\}$ . A realization of the IID1 surrogates can be, e.g.,  $\{b, a, c\}$  and  $\{C, B, A\}$ , while a realization of the IID2 surrogates should look like, e.g.,  $\{b, a, c\}$  and  $\{B, A, C\}$ .

The IID surrogates are suitable in the case of very noisy data, when a narrow band-pass filtering is applied before detecting the phase synchronization. It is important to generate the IID surrogates before the filtering and then to evaluate test statistics from both filtered data and filtered IID surrogates.

3. *FT1 surrogates* are independently generated for each of the two components of the studied bivariate series as realizations of linear stochastic processes with the same sample power spectra as the series under study. The FT1 surrogates are obtained by computing the Fourier transform (FT) of the series, keeping unchanged the magnitudes of the Fourier coefficients (the spectrum), but the phases of the Fourier coefficients are randomized and the inverse FT into the time domain is performed [3, 4]. The FT1 surrogates realize the null hypothesis of two linear stochastic processes which asynchronously oscillate with the same frequencies (power spectra) as the original series under study.

4. *FT2 surrogates* are realizations of a bivariate linear stochastic process which mimics individual spectra of the two components of the original bivariate series as well as their cross-spectrum. Constructing the FT2 surrogates not only the spectra but also differences between phases of the Fourier coefficients of the two series for particular frequency bins must be kept unchanged. Thus the phase randomization is performed by adding the same random number to the phases of both coefficients of the same frequency

bin. (For more details see [7].)

In Figure 1 we present an example of a segment of cardio-respiratory data described in Sec. 2 and related FT1 and FT2 surrogates. All the series are rescaled into zero mean and unit variance, so that the respiratory movement (RM) and the instantaneous heart rate (HR) can be plotted in a common scale. Note the 1:1 locking between RM and HR in the data, which is completely destroyed in the FT1 surrogates, while a coupling in the FT2 surrogates occurs, however, it is somewhat weaker than in the original data (see below).

## 4 Testing nonlinearity

In this section we briefly review a method for detection and characterization of nonlinear relations in multivariate as well as in univariate time series. The method employs the technique of uni- and multivariate surrogate data and information-theoretic functionals called redundancies. The test for nonlinearity based on the redundancy – linear redundancy approach, combined with the surrogate data is described in detail in Ref. [4], its multivariate version in Ref. [8]. The surrogate data, briefly described above, have been introduced in Ref. [3], and the multivariate version in Ref. [7]. More details about the information-theoretic functionals can be found, e.g., in Ref. [9].

Dealing with the problem of detecting nonlinearity in time series it seems reasonable to say when a time series  $\{y(t)\}$  is nonlinear. Intuitively, we can say that it is the case when the relation between  $y(t)$  and  $y(t + \tau)$  is nonlinear, or, cannot be expressed by a linear function. Looking for a more rigorous definition, we use the way we will detect nonlinearity here — by showing that a time series is inconsistent with a linear stochastic process. Thus, for detection of nonlinearity we do not need to exhibit or describe the underlying nonlinear dynamics, but simply to find arguments that a linear model is inadequate. This approach – testing data against a null hypothesis – was advocated by a number of authors [1, 2, 3, 4]. Therefore we need to define a linear stochastic processes.

Let  $\{y(t)\}$  be a time-series realization of a stationary stochastic process  $\{Y(t)\}$ . Without loss of generality we can set its mean to zero.

$\{Y(t)\}$  is a stochastic linear process if  $Y(t)$  can be written as:

$$Y(t) = Y(0) + \sum_{i=1}^{\infty} a(i)Y(t-i) + \sum_{i=0}^{\infty} b(i)N(t-i), \quad (1)$$

where  $b(0) = 1$ ,  $\sum_{i=1}^{\infty} |a(i)| < \infty$ ,  $\sum_{i=0}^{\infty} |b(i)| < \infty$ ,  $\{N(t)\}$  is an independent, identically distributed (iid), normally distributed process with zero mean and finite variance. (For more details see [5].) The requirement for  $\{N(t)\}$  to be an iid process is an important condition, since any stationary process with zero mean possess a Wold decomposition [6] of the form (1), when the process  $\{N(t)\}$  is uncorrelated. The principal difference lies in the fact, that while “independent” means also “uncorrelated”, “uncorrelated” means only “linearly independent” and not independent in general.

Now, having in mind the null hypothesis of a linear stochastic process, we will evaluate a quantitative difference between the data under study and a set of realizations of a linear stochastic process with the same linear properties as the data. If we find evidence that the time series is not consistent with the null hypothesis (of a linear stochastic process), we will consider the series to be nonlinear.

Consider  $n$  discrete random variables  $X_1, \dots, X_n$  with sets of values  $\Xi_1, \dots, \Xi_n$ , respectively. The probability distribution for an individual  $X_i$  is  $p(x_i) = \Pr\{X_i = x_i\}$ ,  $x_i \in \Xi_i$ . We denote the probability distribution function by  $p(x_i)$ , rather than  $p_{X_i}(x_i)$ , for convenience. Analogously, the joint distribution for the  $n$  variables  $X_1, \dots, X_n$  is  $p(x_1, \dots, x_n)$ . The redundancy  $R(X_1; \dots; X_n)$ , in the case of two variables also known as mutual information  $I(X_1; X_2)$ , quantifies average amount of common information, contained in the  $n$  variables  $X_1, \dots, X_n$ :

$$R(X_1; \dots; X_n) = \sum_{x_1 \in \Xi_1} \dots \sum_{x_n \in \Xi_n} p(x_1, \dots, x_n) \log \frac{p(x_1, \dots, x_n)}{p(x_1) \dots p(x_n)}. \quad (2)$$

In addition to the redundancy (2) various types of redundancies can be defined, quantifying the average amounts of information between/among variables or groups of variables. See Ref. [8] for details.

Now, let the  $n$  variables  $X_1, \dots, X_n$  have zero means, unit variances and correlation matrix  $\mathbf{C}$ . Then, we define the *linear redundancy*  $L(X_1; \dots; X_n)$  of  $X_1, X_2, \dots, X_n$  as

$$L(X_1; \dots; X_n) = -\frac{1}{2} \sum_{i=1}^n \log \sigma_i, \quad (3)$$

where  $\sigma_i$  are the eigenvalues of the  $n \times n$  correlation matrix  $\mathbf{C}$ .

If  $X_1, \dots, X_n$  have an  $n$ -dimensional Gaussian distribution, then  $L(X_1; \dots; X_n)$  and  $R(X_1; \dots; X_n)$  (considering the continuous version of (2) [4]) are theoretically equivalent [10].

For any kind of general (nonlinear) redundancy its linear equivalent exists [8]. The general redundancies  $R$  detect all dependences in data under study, while the linear redundancies  $L$  are sensitive only to linear structures. For detailed discussion see Ref. [4].

Having chosen an appropriate type of the redundancy, say  $R(X_1; \dots; X_n)$ , it is possible and useful to investigate dependences among lagged series, i.e., to evaluate redundancies of the type  $R(x_1(t); x_2(t + \tau_1); \dots; x_n(t + \tau_{n-1}))$ . Due to stationarity this redundancy does not depend on the time  $t$  and is a function of the lags  $\tau_1, \dots, \tau_{n-1}$ .

Like in [4] we define the test statistic as the difference between the redundancy obtained for the original data and the mean redundancy of a set of surrogates, in the number of standard deviations (SD's) of the latter. Thus both the redundancies and redundancy-based statistics are functions of the lags  $\tau_1, \dots, \tau_{n-1}$ . When testing nonlinearity in univariate series [4] also redundancies of several variables are evaluated. Those variables, however, are obtained using a one lagged variable with  $\tau_i = i\tau$ , with one variable  $\tau$ . Evaluating the redundancies and related statistics for broad ranges of the lags can bring a problem of simultaneous statistical inference (see [4, 11] and references within for details). This approach, however, can be more reliable than single-valued tests, as it was demonstrated in univariate case in [4]. As far as all the related considerations from [4] directly apply to multivariate problems, we refer readers to Ref. [4] and will not repeat them here, similarly as the discussion of the function of the linear redundancy, which is used to check the quality of the surrogate data: In some cases the surrogates can have auto-/cross-correlations different from the original data, usually due to a numerical artifact. This difference is detected by the redundancies  $R$  (or other nonlinear statistics) and can be erroneously interpreted as detection of nonlinearity in linear data. The linear redundancy-based statistic evaluates just the differences in the “linear properties”, i.e., in autocorrelations in the univariate tests and in crosscorrelations in the multivariate tests. A significant result in linear redundancy-based statistic indicates a problem in the surrogates and necessity of further investigation before a conclusion about (non)linearity of data under study is made. See Refs. [4, 8] for examples.

Another possible source of spurious detection of nonlinearity in a dynamics of a scrutinized time series can be a *static* nonlinearity: Let us suppose that the underlying dynamics of the studied system is linear and Gaussian, i.e., there is an original realization  $\{x_i\}$  of a Gaussian process, but we can measure the series  $\{y_i\}$ ,  $y_i = f(x_i)$ , where  $f$  is a monotonic nonlinear function. Such nonlinearity, which is not intrinsic to the dynamics of the system under study, but can be caused, e.g., by a measurement apparatus, we call static nonlinearity. The static nonlinearity could influence nonlinearity tests, and could eventually be incorrectly interpreted as a nonlinearity in dynamics under study. Therefore, a histogram transformation is usually applied which could eliminate the influence of static nonlinearities. One can transform the data into normal distribution (“Gaussianization”, [4], or adjust the marginal distribution of the surrogates [3]).

A static nonlinear transformation of a time series, or, a static nonlinearity, as it was called above, effectively means that the distribution of the data is not Gaussian (even if the underlying process is Gaussian). Due to the central-limit theorem, the probability distribution of the FT-based surrogate data tends to a Gaussian distribution. The differences in marginal distributions, however, do not have too dramatic effects on the nonlinearity test based on the redundancy (2) when the redundancy is estimated by using equiquantal [4] marginal partitioning of the data, as it was done here (i.e. by using equiprobable marginal bins both the data and surrogates are effectively transformed into a uniform marginal distribution).

In the examples presented here (Fig. 2) the two-variable mutual information  $I(X(t); Y(t + \tau))$  was applied in order to study dynamical relations between the two variables HR and RM. The mutual information  $I(X; Y)[o]$  (and its linear version  $L(X; Y)[o]$ ) from the scrutinized data and the mean mutual information  $I(X; Y)[s]$  ( $L(X; Y)[s]$ ) from the surrogates, as well as the test statistics (Fig. 2d), defined above, were plotted as functions of lag  $\tau$ . Lack of significant differences between  $L(X; Y)[o]$  and  $L(X; Y)[s]$  (Fig. 2a, and the dashed line in Fig. 2d) confirms the preservation of HR-RM (linear) cross-correlation in the bivariate (FT2) surrogates, while the significant differences found between  $I(X; Y)[o]$  and  $I(X; Y)[s]$  (Fig. 2c, and the solid line in Fig. 2d) infer nonlinearity (i.e., more cross-dependence than just the linear cross-correlation) in the relation between HR and RM. Note that the FT1 surrogates (Fig. 2c, thin lines) do not contain any cross-dependence, i.e.  $I(X; Y)[s]$  vanishes in this case.

## 5 Detection of Phase Synchronization

In the classical case of *periodic* self-sustained oscillators, *phase* synchronization is usually defined as locking of phases  $\phi_{1,2}$ :

$$n\phi_1 - m\phi_2 = \text{const.}, \quad (4)$$

for integer  $n$  and  $m$ , while the amplitudes can be different. Recently, Rosenblum et al. [12] have discovered the phase synchronization in a case of coupled *chaotic* systems, where the phase entrainment (locking) is described as

$$|n\phi_1 - m\phi_2| < \text{const.}, \quad (5)$$

while the amplitudes of the two systems may be completely uncorrelated, i.e., linearly independent. The following considerations are valid for chaotic as well as stochastic oscillators [13].

The phase of a signal  $s(t)$  can be determined by using the analytic signal concept of Gabor [14]. The analytic signal  $\psi(t)$  is a complex function of time defined as

$$\psi(t) = s(t) + j\hat{s}(t) = A(t)e^{j\phi(t)}, \quad (6)$$

where the function  $\hat{s}(t)$  is the Hilbert transform of  $s(t)$

$$\hat{s}(t) = \frac{1}{\pi} \text{P.V.} \int_{-\infty}^{\infty} \frac{s(\tau)}{t - \tau} d\tau. \quad (7)$$

(P.V. means that the integral is taken in the sense of the Cauchy principal value.) The instantaneous phase  $\phi(t)$  of the signal  $s(t)$  is then

$$\phi(t) = \arctan \frac{\hat{s}(t)}{s(t)}. \quad (8)$$

By computing and plotting the phase difference  $\Delta\phi = m\phi_1 - n\phi_2$ , the phase synchronization was observed in chaotic systems modelled on digital [12] or analog [15] computers, as well as in bivariate experimental data such as those registered from a mammalian cardio-respiratory system [16]. In general, however, the evaluation of the instantaneous phase difference  $\Delta\phi(t)$  alone can be insufficient for answering the question about existence of the phase synchronization between (sub)systems under study. Paluš [17] has demonstrated that the instantaneous phases  $\phi_1, \phi_2$  of phase-synchronized (sub)systems are confined in strip-like structures, when plotted in the plane  $(-\pi, \pi) \times (-\pi, \pi)$ , while the instantaneous phases  $\phi_1, \phi_2$  of asynchronous processes almost homogeneously fill the plane  $(-\pi, \pi) \times (-\pi, \pi)$ . This phenomenon is demonstrated in Figs 3a, c, d, where the instantaneous phases  $\phi_1, \phi_2$  obtained from HR and RM series (Fig. 3a), as well  $\phi_1, \phi_2$  from related FT2 (Fig. 3c) and FT1 (Fig. 3d) surrogates are plotted. The dependence between  $\phi_1$  and  $\phi_2$ , i.e., the structure in the  $\phi_1, \phi_2$  plot, can again be quantified by using the mutual information, now defined as

$$I(\phi_1, \phi_2) = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} p_{1,2}(\phi_1, \phi_2) \log \frac{p_{1,2}(\phi_1, \phi_2)}{p_1(\phi_1)p_2(\phi_2)} d\phi_1 d\phi_2, \quad (9)$$

where  $p_1(\phi_1)$  and  $p_2(\phi_2)$  are probability distributions of the phases  $\phi_1$  and  $\phi_2$ , respectively, and  $p_{1,2}(\phi_1, \phi_2)$  is their joint distribution. Theoretically, independence of the phases, i.e., the absence of the phase synchronization means homogeneous distribution of the points  $(\phi_1, \phi_2)$  and  $I(\phi_1, \phi_2) = 0$ ; while for the phase synchronization, i.e., a mutual dependence of the phases,  $I(\phi_1, \phi_2) > 0$  holds. For detecting the phase

synchronization in experimental data it is necessary to establish that  $I(\phi_1, \phi_2) > 0$  with a statistical significance. Therefore we again use the surrogate data technique, in particular, we compare  $I(\phi_1, \phi_2)$  obtained from experimental data with a range of  $I(\phi_1, \phi_2)$  obtained from a set of appropriate surrogates, as defined above. In order to obtain an evidence for the phase synchronization it is necessary to reject the IID1, IID2 and FT1 surrogate null hypotheses (see [17] and Sec. 3 for details). In the presented example of the cardio-respiratory synchronization (See Fig. 1 for example of the data and FT1,2 surrogates, and Figs. 3a,c,d for the  $(\phi_1, \phi_2)$  plots for the data, FT1 and FT2 surrogates, respectively) it is trivial to reject the IID surrogates. Figure 3b demonstrates the rejection of both the FT1 and FT2 surrogate nulls. As noted above, the rejection of the FT1 null (no synchronization) can be interpreted as a positive identification of the phase synchronization in the studied cardiorespiratory data (HR, RM). The FT2 surrogates preserve (a part of) synchronization, if present in the original data (see Figs. 1c, 3c), which is consistent with the hypothesis of a bivariate linear stochastic process. The rejection of the FT2 null (Fig. 3b, the squares represent  $I(\phi_1, \phi_2)$  from a set of 30 realization of the FT2 surrogates) confirms the fact, that the cardio-respiratory synchronization is a nonlinear phenomenon, for which a linear stochastic explanation is insufficient.

## 6 Conclusion

Methods for detecting nonlinearity in univariate and multivariate time series, and for identification of phase synchronization in bivariate time series has been presented, which both utilized the technique of surrogate data. The detection of nonlinearity was based on rejecting a null hypothesis of a linear stochastic process with the same spectra and cross-spectra as the studied series, while the phase synchronization was identified by rejecting a series of hypotheses represented by the surrogate data without any cross-dependence (IID1 – independent noises, FT1 – asynchronous oscillations), or with possible trivial cross-dependence (not the phase synchronization, IID2). The FT2 surrogates represent the special case of possible linear stochastic synchronization, which is rejected in the case of phase-synchronized nonlinear systems.

Note that the two above methods are not equivalent neither in general, nor in the case of common rejection of the FT2 null. The detection of nonlinearity, based on rejection of the null hypothesis of a bi-(multi)variate linear stochastic process indicates presence of an unspecified nonlinear link between the variables, while the second method attempts to identify very specific relation of the phase synchronization. The latter method is a new one and is still under intensive development. In the nonlinearity tests the results can be influenced by changes of amplitude distribution caused by the phase-randomized FT surrogates. Therefore certain histogram transformations are applied [4, 18]. The situation is different when testing the phase synchronization, since the phases of asynchronous oscillators as well as the phases of the asynchronous FT1 surrogates homogeneously fill the plane  $(-\pi, \pi) \times (-\pi, \pi)$ . Therefore the rejection of the asynchronous null hypothesis is reliable if enough data are available. For the cases of short time series further research is desirable to establish the reliability of the test.

The application of the above described methods to the data registered from a newborn piglet during quiet sleep made possible to demonstrate that the interactions between respiratory movements and heart rate fluctuations possess a significant nonlinearity and involve certain phase synchronizations.

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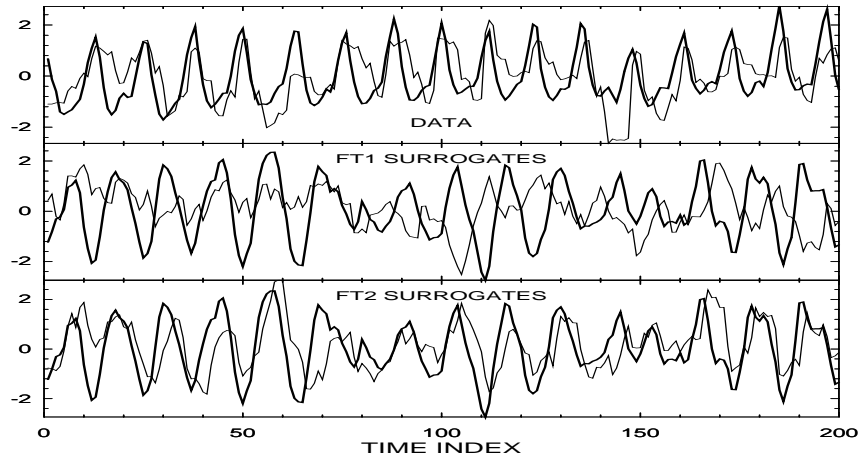


Figure 1: The example of a segment of the heart rate (HR, thin lines) and respiratory movements (RM, solid lines) data (upper panel) and related FT1 (middle panel) and FT2 (lower panel) surrogate realizations. The data and the surrogates are centered and rescaled in order to have zero mean and unit variance.

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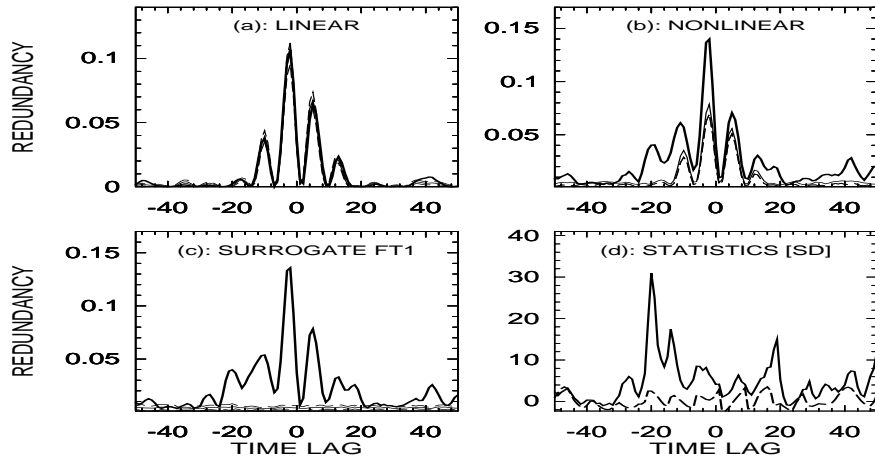


Figure 2: a, b, c: The mutual information (redundancy) – a: the linear version  $L(X;Y)$ , b, c: the general – nonlinear version  $I(X;Y)$  between the HR and RM series (solid lines) and related surrogates (mean and mean  $\pm$  SD of a set of 30 surrogate realizations, plotted by using thin full and thin dashed lines respectively) as a function of time lag. The FT2 surrogates are used in the multivariate nonlinearity tests (a, b), the FT1 surrogates are illustrated in plot c. d: The linear (dashed line) and nonlinear (solid line) test statistics, in number of standard deviations, related to plots a and b, respectively.



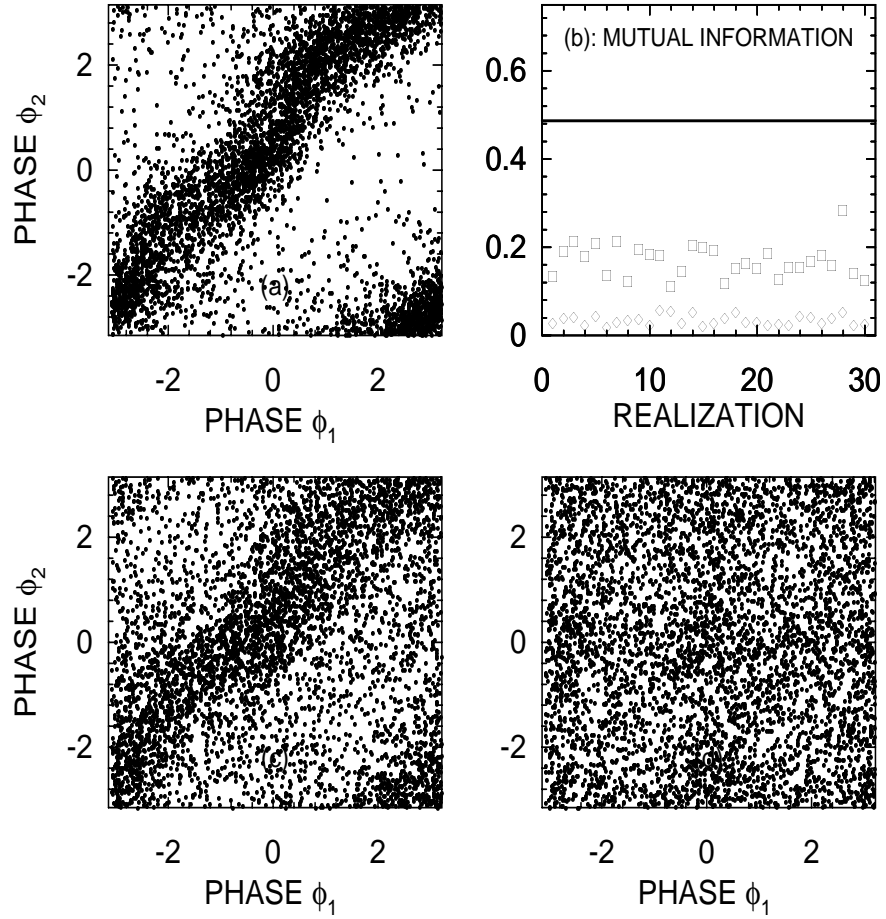


Figure 3: The plot of the instantaneous phases  $\phi_1$  and  $\phi_2$  obtained from the HR and RM series, respectively (a – upper left plot) and from a realization of the FT2 (c – lower left plot) and FT1 (d – lower right plot) surrogate data. The mutual information  $I(\phi_1, \phi_2)$  (b – upper right plot) for the phases of HR and RM (the solid line) and  $I(\phi_1, \phi_2)$  for the phases of 30 realizations of FT1 (diamonds) and FT2 (squares) surrogates.