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Modified CUTE Problems for Sparse Unconstrained Optimization

Ladislav Lukšan, Ctirad Matonoha, Jan Vlček

Technical report No. 1081

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Pod Vodárenskou věží 2, 182 07 Prague 8 phone: +420 2 688 42 44, fax: +420 2 858 57 89,
e-mail:e-mail:ics@cs.cas.cz

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Abstract:

This report contains a description of subroutines which can be used for testing unconstrained optimization codes. These subroutines can easily be obtained either by using the anonymous ftp address `ftp://ftp.cs.cas.cz/pub/msdos/opt` (file `TEST11.FOR`) or from the web homepage `http://www.cs.cas.cz/luksan/test.html`. Furthermore, all test problems contained in these subroutines are presented in the analytic form.

Keywords:
unconstrained optimization, test problems

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1 Introduction

This report describes subroutines `TIUX11`, `TIUD11`, `TFFU11`, `TFGU11` `TFBU11` which contain 82 sparse problems for testing unconstrained optimization codes. These problems were chosen from the CUTE testing environment [2] (we have selected only problems for large-scale sparse unconstrained optimization with an arbitrary number of variables). Most of these problems were implemented using the AMPL sources contained in [1]. Several CUTE problems were modified:

- We added four new problems DIXMAANM, DIXMAANN, DIXMAANO, DIXMAANP.
- Original problems CHNROSNB and ERRINROS use parameters α_i, $1 \le i \le n$ corresponding to $n = 50$. We have replaced these particular parameters, lying between 0.5 and 2.5, by analytically defined parameters $\alpha_i = 1.5 + \sin(i)$, which allows us to choose an arbitrary dimension of these problems.
- Since problems FLETCBV3 and INDEF are not bounded from below, we have replaced their linear terms x_i, $1 \le i \le n$ by nonlinear terms $100 sin(x_i/100)$, $1 \le i \le n$.
- The SIF files of NCB20 and NCB20B contain misprints. We have corrected them.
- Problems SBRYBND and SCOSINE contain too large scaling factor. We have changed the scaling factor from 12 to 6.

Our implementation of the selected CUTE problems was motivated by the requirement that all problems should be tested simultaneously in the same run.

All subroutines are written in the standard Fortran 77 language. Their names are derived from the following rule:

- The first letter is `T` - test subroutines.
- The second letter is either `I` - initiation, or `F` - objective function.
- The third letter is either `U` - initiation of unconstrained problem, or `F` - computation of the function value, or `G` - computation of the gradient vector, or `B` - computation of both the function value and the gradient vector simultaneously.
- The fourth letter is either `U` - universal subroutine, or `D` - subroutine for problems with unknown sparsity patterns, or `X` - an artificial subroutine (defining the dimensions of problems).

The last two digits determine a given collection (the numbering corresponds to the UFO system [3], which contains similar collections).

Initiation subroutines use the following parameters (array dimensions are given in parentheses):

`N`	input	number of variables,
`M`	output	number of elements of the sparse Hessian matrix,
`X(N)`	output	vector of variables,
`IH(N+1)`	output	pointers of diagonal elements of the sparse Hessian matrix,
`JH(M)`	output	column indices of nonzero elements of the sparse Hessian matrix,
`FMIN`	output	lower bound of the objective function value,
`XMAX`	output	maximum stepsize,
`NEXT`	input	number of the problem selected,
`IERR`	output	error indicator (0 - correct data, 1 - `N` is too small).

Although `N` is an input parameter, it can be changed by the initiation subroutine when its value does not satisfy the required conditions. For example, most of the problems require `N` to be even or a multiple of a positive integer.

Evaluation subroutines use the following parameters (array dimensions are given in parentheses):

`N`	input	number of variables,
`X(N)`	input	vector of variables,
`F`	output	value of the objective function,
`G(N)`	output	gradient of the objective function,
`NEXT`	input	number of the problem selected.

2 Test problems for general unconstrained optimization

Calling statements have the form

```
CALL TIUX11(N,NEXT)
CALL TIUD11(N,X,FMIN,XMAX,NEXT,IERR)
CALL TFFU11(N,X,F,NEXT)
CALL TFGU11(N,X,G,NEXT)
CALL TFBU11(N,X,F,G,NEXT)
```

with the following significance:

- `TIUX11` – selection of dimension `N`.
- `TIUD11` – initiation of vector of variables `X`, which has dimension `N`.
- `TFFU11` – evaluation of the general objective function value `F` at the point `X`.
- `TFGU11` – evaluation of the general objective function gradient `G` at the point `X`.
- `TFBU11` – evaluation of the general objective function value `F` and gradient `G` at the point `X`.

We seek a minimum of a general objective function $F(x)$ from the starting point $\bar{x}$. For positive integers k and l, we use the notation $\mathrm{div}(k,l)$ for integer division, i.e., maximum integer not greater than k/l, and $\mathrm{mod}(k,l)$ for the remainder after integer division, i.e., $\mathrm{mod}(k,l) = l(k/l - \mathrm{div}(k,l))$. The description of individual problems follows.

Problem 1. ARWHEAD

$$F(x) = \sum_{i=1}^{n-1} \left[(x_i^2 + x_n^2)^2 - 4x_i + 3 \right],$$

$$\bar{x}_i = 1.0, \quad i \geq 1.$$

Problem 2. BDQRTIC

$$F(x) = \sum_{i=1}^{n-4} \left[(3 - 4x_i)^2 + (x_i^2 + 2x_{i+1}^2 + 3x_{i+2}^2 + 4x_{i+3}^2 + 5x_n^2)^2 \right],$$

$$\bar{x}_i = 1.0, \quad i \geq 1.$$

Problem 3. BROYDN7D

$$F(x) = \sum_{i=1}^{n} |1 - x_{i-1} - 2x_{i+1} + (3 - 2x_i)x_i|^p + \sum_{i=1}^{n/2} |x_i + x_{i+n/2}|^p,$$

$$p = 7/3, \quad x_0 = x_{n+1} = 0, \quad \bar{x}_i = 1.0, \quad i \geq 1.$$

Problem 4. BRYBND

$$F(x) = \sum_{i=1}^{n} \left[x_i(2 + 5x_i^2) + 1 - \sum_{j \in J_i} x_j(1 + x_j) \right]^2,$$

$$J_i = \{j : j \neq i,\ \max(1, i-5) \leq j \leq \min(n, i+1)\}, \quad i = 1, \ldots, n,$$

$$\bar{x}_i = -1.0, \quad i \geq 1.$$

Problem 5. CHAINWOO

$$\begin{aligned} F(x) = 1 \quad &+ \quad \sum_{i=1}^{n/2-1} \Big[100(x_{2i} - x_{2i-1}^2)^2 + (1 - x_{2i-1})^2 + 90(x_{2i+2} - x_{2i+1}^2)^2 \\ &+ \quad (1 - x_{2i+1})^2 + 10(x_{2i} + x_{2i+2} - 2)^2 + \frac{1}{10}(x_{2i} - x_{2i+2})^2 \Big], \end{aligned}$$

$$\bar{x}_1 = \bar{x}_3 = -3.0, \quad \bar{x}_2 = \bar{x}_4 = -1.0, \quad \bar{x}_i = -2.0, \quad i \geq 5.$$

Problem 6. COSINE

$$F(x) = \sum_{i=1}^{n-1} \cos(x_i^2 - \frac{1}{2} x_{i+1}),$$

$$\bar{x}_i = 1.0, \quad i \geq 1.$$

Problem 7. CRAGGLVY

$$\begin{aligned} F(x) \quad &= \quad \sum_{i=1}^{n/2-1} \Big[[\exp(x_{2i-1}) - x_{2i}]^4 + 100(x_{2i} - x_{2i+1})^6 \\ &+ \quad [\tan(x_{2i+1} - x_{2i+2}) + x_{2i+1} - x_{2i+2}]^4 + x_{2i-1}^8 + (x_{2i+2} - 1)^2 \Big], \end{aligned}$$

$$\bar{x}_1 = 1.0, \quad \bar{x}_i = 2.0, \quad i \geq 2.$$

Problem 8. CURLY10

$$F(x) = \sum_{i=1}^{n} f_i(x)[f_i(x)(f_i^2(x) - 20) - 0.1],$$

$$f_i(x) = \sum_{j=i}^{\min(i+k,n)} x_j, \quad k = 10, \quad \bar{x}_i = 0.0001/(n+1).$$

Problem 9. CURLY20

$$F(x) = \sum_{i=1}^{n} f_i(x)[f_i(x)(f_i^2(x) - 20) - 0.1],$$

$$f_i(x) = \sum_{j=i}^{\min(i+k,n)} x_j, \quad k = 20, \quad \bar{x}_i = 0.0001/(n+1).$$

Problem 10. CURLY30

$$F(x) = \sum_{i=1}^{n} f_i(x)[f_i(x)(f_i^2(x) - 20) - 0.1],$$

$$f_i(x) = \sum_{j=i}^{\min(i+k,n)} x_j, \quad k = 30, \quad \bar{x}_i = 0.0001/(n+1).$$

Problem 11. DIXMAANE

$$F(x) = 1 + \sum_{i=1}^{n} \frac{i}{n}\alpha x_i^2 + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 + \sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 + \sum_{i=1}^{m} \frac{i}{n}\delta x_i x_{i+2m},$$

$$m = n/3, \quad \alpha = 1.0, \quad \beta = 0.0, \quad \gamma = 0.125, \quad \delta = 0.125,$$

$$\bar{x}_i = 2.0, \quad i \geq 1.$$

Problem 12. DIXMAANF

$$F(x) = 1 + \sum_{i=1}^{n} \frac{i}{n}\alpha x_i^2 + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 + \sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 + \sum_{i=1}^{m} \frac{i}{n}\delta x_i x_{i+2m},$$

$$m = n/3, \quad \alpha = 1.0, \quad \beta = 0.0625, \quad \gamma = 0.0625, \quad \delta = 0.0625,$$

$$\bar{x}_i = 2.0, \quad i \geq 1.$$

Problem 13. DIXMAANG

$$F(x) = 1 + \sum_{i=1}^{n} \frac{i}{n}\alpha x_i^2 + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 + \sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 + \sum_{i=1}^{m} \frac{i}{n}\delta x_i x_{i+2m},$$

$$m = n/3, \quad \alpha = 1.0, \quad \beta = 0.125, \quad \gamma = 0.125, \quad \delta = 0.125,$$

$$\bar{x}_i = 2.0, \quad i \geq 1.$$

Problem 14. DIXMAANH

$$F(x) = 1 + \sum_{i=1}^{n} \frac{i}{n}\alpha x_i^2 + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 + \sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 + \sum_{i=1}^{m} \frac{i}{n}\delta x_i x_{i+2m},$$

$$m = n/3, \quad \alpha = 1.0, \quad \beta = 0.26, \quad \gamma = 0.26, \quad \delta = 0.26,$$

$$\bar{x}_i = 2.0, \quad i \geq 1.$$

Problem 15. DIXMAANI

$$F(x) = 1 + \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \alpha x_i^2 + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 + \sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 + \sum_{i=1}^{m} \left(\frac{i}{n}\right)^2 \delta x_i x_{i+2m},$$

$$m = n/3, \quad \alpha = 1.0, \quad \beta = 0.0, \quad \gamma = 0.125, \quad \delta = 0.125,$$

$$\bar{x}_i = 2.0, \quad i \geq 1.$$

Problem 16. DIXMAANJ

$$F(x) = 1 + \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \alpha x_i^2 + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 + \sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 + \sum_{i=1}^{m} \left(\frac{i}{n}\right)^2 \delta x_i x_{i+2m},$$

$$m = n/3, \quad \alpha = 1.0, \quad \beta = 0.0625, \quad \gamma = 0.0625, \quad \delta = 0.0625,$$

$$\bar{x}_i = 2.0, \quad i \geq 1.$$

Problem 17. DIXMAANK

$$F(x) = 1 + \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \alpha x_i^2 + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 + \sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 + \sum_{i=1}^{m} \left(\frac{i}{n}\right)^2 \delta x_i x_{i+2m},$$

$$m = n/3, \quad \alpha = 1.0, \quad \beta = 0.125, \quad \gamma = 0.125, \quad \delta = 0.125,$$

$$\bar{x}_i = 2.0, \quad i \geq 1.$$

Problem 18. DIXMAANL

$$F(x) = 1 + \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \alpha x_i^2 + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 + \sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 + \sum_{i=1}^{m} \left(\frac{i}{n}\right)^2 \delta x_i x_{i+2m},$$

$$m = n/3, \quad \alpha = 1.0, \quad \beta = 0.26, \quad \gamma = 0.26, \quad \delta = 0.26,$$

$$\bar{x}_i = 2.0, \quad i \geq 1.$$

Problem 19. DIXMAANM

$$F(x) = 1 + \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \alpha x_i^2 + \sum_{i=1}^{n-1} \frac{i}{n} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 + \sum_{i=1}^{2m} \frac{i}{n} \gamma x_i^2 x_{i+m}^4 + \sum_{i=1}^{m} \left(\frac{i}{n}\right)^2 \delta x_i x_{i+2m},$$

$$m = n/3, \quad \alpha = 1.0, \quad \beta = 0.0, \quad \gamma = 0.125, \quad \delta = 0.125,$$

$$\bar{x}_i = 2.0, \quad i \geq 1.$$

Problem 20. DIXMAANN

$$F(x) = 1 + \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \alpha x_i^2 + \sum_{i=1}^{n-1} \frac{i}{n} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 + \sum_{i=1}^{2m} \frac{i}{n} \gamma x_i^2 x_{i+m}^4 + \sum_{i=1}^{m} \left(\frac{i}{n}\right)^2 \delta x_i x_{i+2m},$$

$$m = n/3, \quad \alpha = 1.0, \quad \beta = 0.0625, \quad \gamma = 0.0625, \quad \delta = 0.0625,$$

$$\bar{x}_i = 2.0, \quad i \geq 1.$$

Problem 21. DIXMAANO

$$F(x) = 1 + \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \alpha x_i^2 + \sum_{i=1}^{n-1} \frac{i}{n} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 + \sum_{i=1}^{2m} \frac{i}{n} \gamma x_i^2 x_{i+m}^4 + \sum_{i=1}^{m} \left(\frac{i}{n}\right)^2 \delta x_i x_{i+2m},$$

$$m = n/3, \quad \alpha = 1.0, \quad \beta = 0.125, \quad \gamma = 0.125, \quad \delta = 0.125,$$

$$\bar{x}_i = 2.0, \quad i \geq 1.$$

Problem 22. DIXMAANP

$$F(x) = 1+\sum_{i=1}^{n}\left(\frac{i}{n}\right)^2\alpha x_i^2+\sum_{i=1}^{n-1}\frac{i}{n}\beta x_i^2(x_{i+1}+x_{i+1}^2)^2+\sum_{i=1}^{2m}\frac{i}{n}\gamma x_i^2 x_{i+m}^4+\sum_{i=1}^{m}\left(\frac{i}{n}\right)^2\delta x_i x_{i+2m},$$

$$m = n/3, \quad \alpha = 1.0, \quad \beta = 0.26, \quad \gamma = 0.26, \quad \delta = 0.26,$$

$$\bar{x}_i = 2.0, \quad i \geq 1.$$

Problem 23. DQRTIC

$$F(x) = \sum_{i=1}^{n}(x_i - i)^4,$$

$$\bar{x}_i = 2.0, \quad i \geq 1.$$

Problem 24. EDENSCH

$$F(x) = 16 + \sum_{i=1}^{n-1}\Big[(x_i - 2)^4 + (x_i x_{i+1} - 2x_{i+1})^2 + (x_{i+1} + 1)^2\Big],$$

$$\bar{x}_i = 0.0, \quad i \geq 1.$$

Problem 25. EG2

$$F(x) = \sum_{i=1}^{n-1}\sin(x_1 + x_i^2 - 1) + \frac{1}{2}\sin(x_n^2),$$

$$\bar{x}_i = 0.0, \quad i \geq 1.$$

Problem 26. ENGVAL1

$$F(x) = \sum_{i=1}^{n-1}\Big[(x_i^2 + x_{i+1}^2)^2 - 4x_i + 3\Big],$$

$$\bar{x}_i = 2.0, \quad i \geq 1.$$

Problem 27. CHNROSNB – modified

$$F(x) = \sum_{i=2}^{n}\Big[16(x_{i-1} - x_i^2)^2(1.5 + \sin(i))^2 + (x_i - 1)^2\Big],$$

$$\bar{x}_i = -1.0, \quad i \geq 1.$$

Problem 28. ERRINROS – modified

$$F(x) = \sum_{i=2}^{n}\Big[[x_{i-1} - 16x_i^2(1.5 + \sin(i))^2]^2 + (x_i - 1)^2\Big],$$

$$\bar{x}_i = -1.0, \quad i \geq 1.$$

Problem 29. EXTROSNB

$$F(x) = (x_1 - 1)^2 + 100 \sum_{i=2}^{n} (x_i - x_{i-1}^2)^2,$$

$$\bar{x}_i = 1.0, \quad i \geq 1.$$

Problem 30. FLETCBV3 – modified

$$F(x) = \frac{p}{2} \Big[x_1^2 + \sum_{i=1}^{n-1} (x_i - x_{i+1})^2 + x_n^2 \Big] - p \sum_{i=1}^{n} \Big[100 \left(1 + \frac{2}{h^2} \right) \sin \left(\frac{x_i}{100} \right) + \frac{1}{h^2} \cos x_i \Big],$$

$$p = 10^{-8}, \quad h = 1/(n+1), \quad \bar{x}_i = i/(n+1), \quad i = 1, \ldots, n.$$

Problem 31. FLETCBV2

$$F(x) = \frac{1}{2} \Big[x_1^2 + \sum_{i=1}^{n-1} (x_i - x_{i+1})^2 + x_n^2 \Big] - h^2 \sum_{i=1}^{n} (2x_i + \cos x_i) - x_n,$$

$$h = 1/(n+1), \quad \bar{x}_i = i/(n+1), \quad i = 1, \ldots, n.$$

Problem 32. FLETCHCR

$$F(x) = 100 \sum_{i=1}^{n-1} (x_{i+1} - x_i + 1 - x_i^2)^2,$$

$$\bar{x}_i = 0.0, \quad i \geq 1.$$

Problem 33. FMINSRF2

$$F(x) = \frac{y_{p/2,p/2}^2}{n} + \frac{1}{(p-1)^2} \sum_{i=1}^{p-1} \sum_{j=1}^{p-1} \sqrt{1 + \frac{1}{2}(p-1)^2 \Big[(y_{i,j} - y_{i+1,j+1})^2 + (y_{i+1,j} - x_{i,j+1})^2 \Big]},$$

$$y_{i,j} = x_{(i-1)p+j}, \quad 1 \leq i \leq p, \quad 1 \leq j \leq p, \quad p = \sqrt{n},$$

$$\bar{x}_{(i-1)p+j} = \bar{y}_{i,j}, \quad 1 \leq i \leq p, \quad 1 \leq j \leq p,$$

$$\bar{y}_{1,j} = 4(j-1)/(p-1) + 1, \quad \bar{y}_{p,j} = 4(j-1)/(p-1) + 5, \quad 1 \leq j \leq p,$$

$$\bar{y}_{i,p} = 8(i-1)/(p-1) + 1, \quad \bar{y}_{i,1} = 8(i-1)/(p-1) + 9, \quad 1 \leq i \leq p,$$

$$\bar{y}_{i,j} = 0, \quad 1 < i < p, \quad 1 < j < p.$$

Problem 34. FREUROTH

$$F(x) = \sum_{i=1}^{n-1} \Big[(5 - x_{i+1}) x_{i+1}^2 + x_i - 2x_{i+1} - 13 \Big]^2 + \sum_{i=1}^{n-1} \Big[(1 + x_{i+1}) x_{i+1}^2 + x_i - 14x_{i+1} - 29 \Big]^2,$$

$$\bar{x}_1 = 0.5, \quad \bar{x}_2 = -2.0, \quad \bar{x}_i = 0.0, \quad i \geq 3.$$

Problem 35. GENHUMPS

$$F(x) = \sum_{i=1}^{n-1} \Big[\sin^2(20x_i)\sin^2(20x_{i+1}) + \frac{1}{20}(x_i^2 + x_{i+1}^2) \Big],$$

$$\bar{x}_1 = -506.0, \quad \bar{x}_i = -506.2, \quad i \geq 2.$$

Problem 36. GENROSE

$$F(x) = 1 + \sum_{i=2}^{n} \Big[100(x_i - x_{i-1}^2)^2 + (x_i - 1)^2 \Big],$$

$$\bar{x}_i = i/(n+1), \quad i \geq 1.$$

Problem 37. INDEF – modified

$$F(x) = 100 \sum_{i=1}^{n} \sin\Big(\frac{x_i}{100}\Big) + \frac{1}{2}\sum_{i=2}^{n-1} \cos(2x_i - x_n - x_1),$$

$$\bar{x}_i = i/(n+1), \quad i \geq 1.$$

Problem 38. LIARWHD

$$F(x) = \sum_{i=1}^{n} \Big[4(x_i^2 - x_1)^2 + (x_i - 1)^2 \Big],$$

$$\bar{x}_i = 4.0, \quad i \geq 1.$$

Problem 39. MOREBV – different start point

$$F(x) = \sum_{i=1}^{n} \Big[2x_i - x_{i-1} - x_{i+1} + \frac{h^2}{2}(x_i + ih + 1)^3 \Big]^2,$$

$$h = 1/(n+1), \quad x_0 = x_{n+1} = 0, \quad \bar{x}_i = 0.5, \quad i \geq 1.$$

Problem 40. NCB20 – corrected

$$\begin{aligned} F(x) &= 2 + \sum_{i=1}^{n-30} \Big[\frac{10}{i}\Big(\sum_{j=1}^{20} \frac{x_{i+j-1}}{1 + x_{i+j-1}^2} \Big)^2 - \frac{1}{5}\sum_{j=1}^{20} x_{i+j-1} \Big] \\ &+ \sum_{i=1}^{n-10} (x_i^4 + 2) + 10^{-4} \sum_{i=1}^{10} (x_i x_{i+10} x_{i+n-10} + 2x_{i+n-10}^2), \end{aligned}$$

$$\bar{x}_i = 0.0, \quad 1 \leq i \leq n-10, \quad \bar{x}_i = 1.0, \quad n-10 < i \leq n.$$

Problem 41. NCB20B – corrected

$$F(x) = \sum_{i=1}^{n-19} \Big[\frac{10}{i}\Big(\sum_{j=1}^{20} \frac{x_{i+j-1}}{1 + x_{i+j-1}^2} \Big)^2 - \frac{1}{5}\sum_{j=1}^{20} x_{i+j-1} \Big] + \sum_{i=1}^{n} (100x_i^4 + 2),$$

$$\bar{x}_i = 0.0, \quad i \geq 1.$$

Problem 42. NONCVXUN

$$F(x) = \sum_{i=1}^{n} \Big[(x_i + x_{j+1} + x_{k+1})^2 + 4\cos(x_i + x_{j+1} + x_{k+1}) \Big],$$

$$j = \mathrm{mod}(2i-1, n), \quad k = \mathrm{mod}(3i-1, n), \quad \bar{x}_i = i, \quad i \geq 1.$$

Problem 43. NONCVXU2

$$F(x) = \sum_{i=1}^{n} \Big[(x_i + x_{j+1} + x_{k+1})^2 + 4\cos(x_i + x_{j+1} + x_{k+1}) \Big],$$

$$j = \mathrm{mod}(3i-2, n), \quad k = \mathrm{mod}(7i-3, n), \quad \bar{x}_i = i, \quad i \geq 1.$$

Problem 44. NONDIA

$$F(x) = (x_1 - 1)^2 + \sum_{i=2}^{n} (100x_1 - x_{i-1}^2)^2,$$

$$\bar{x}_i = -1.0, \quad i \geq 1.$$

Problem 45. NONDQUAR

$$F(x) = (x_1 - x_2)^2 + (x_{n-1} - x_n)^2 + \sum_{i=1}^{n-2} (x_i + x_{i+1} + x_n)^4,$$

$$\bar{x}_i = 1.0, \quad \mathrm{mod}(i,2) = 1, \quad \bar{x}_i = -1.0, \quad \mathrm{mod}(i,2) = 0.$$

Problem 46. PENALTY3

$$\begin{aligned} F(x) &= 1 + \sum_{i=1}^{n/2} (x_i - 1)^2 + \exp(x_n) \sum_{i=1}^{n-2} (x_i + 2x_{i+1} + 10x_{i+2} - 1)^2 \\ &+ \sum_{i=1}^{n-2} (x_i + 2x_{i+1} + 10x_{i+2} - 1)^2 \sum_{i=1}^{n-2} (2x_i + x_{i+1} - 3)^2 \\ &+ \exp(x_{n-1}) \sum_{i=1}^{n-2} (2x_i + x_{i+1} - 3)^2 + \Big[\sum_{i=1}^{n} (x_i^2 - n) \Big]^2, \end{aligned}$$

$$\bar{x}_i = i/(n+1), \quad i \geq 1.$$

Problem 47. POWELLSG

$$F(x) = \sum_{i=1}^{n/4} (x_j + 10x_{j+1})^2 + 5(x_{j+2} - x_{j+3})^2 + (x_{j+1} - 2x_{j+2})^4 + 10(x_j - x_{j+3})^4,$$

$$j = 4(i-1) + 1,$$

$$\bar{x}_i = 3.0, \quad \mathrm{mod}(i,4) = 1, \quad \bar{x}_i = -1.0, \quad \mathrm{mod}(i,4) = 2,$$

$$\bar{x}_i = 0.0, \quad \mathrm{mod}(i,4) = 3, \quad \bar{x}_i = 1.0, \quad \mathrm{mod}(i,4) = 0.$$

Problem 48. SBRYBND – different scaling

$$F(x) = \sum_{i=1}^{n} \Big[(2 + 5p_i^2 x_i^2) p_i x_i + 1 - \sum_{j \in J_i} p_j x_j (1 + p_j x_j) \Big],$$

$$J_i = \{ j : j \neq i,\ \max(1, i-5) \le j \le \min(n, i+1) \}, \quad i = 1, \dots, n,$$

$$\bar{x}_i = 1/p_i, \quad i \ge 1, \quad p_i = \exp\Big(6 \frac{i-1}{n-1} \Big).$$

Problem 49. SCHMVETT

$$F(x) = \sum_{i=1}^{n-2} \Big[- \frac{1}{1 + (x_i - x_{i+1})^2} - \sin\Big(\frac{\pi x_{i+1} + x_{i+2}}{2} \Big) - \exp\Big[- \Big(\frac{x_i + x_{i+2}}{x_{i+1}} - 2 \Big)^2 \Big] \Big],$$

$$\bar{x}_i = 3.0, \quad i \ge 1.$$

Problem 50. SCOSINE – different scaling

$$F(x) = \sum_{i=1}^{n-1} \cos\Big(p_i^2 x_i^2 - \frac{p_{i+1} x_{i+1}}{2} \Big),$$

$$\bar{x}_i = 1/p_i, \quad i \ge 1, \quad p_i = \exp\Big(6 \frac{i-1}{n-1} \Big).$$

Problem 51. SINQUAD

$$F(x) = (x_1 - 1)^4 + (x_n^2 - x_1^2)^2 + \sum_{i=2}^{n-1} \Big[\sin(x_i - x_n) - x_1^2 + x_i^2 \Big]^2,$$

$$\bar{x}_i = 0.1, \quad i \ge 1.$$

Problem 52. SPARSINE

$$F(x) = \frac{1}{2} \sum_{i=1}^{n} i \Big[\sin x_i + \sin x_{j_1} + \sin x_{j_2} + \sin x_{j_3} + \sin x_{j_4} + \sin x_{j_5} \Big]^2,$$

$$j_1 = \mathrm{mod}(2i - 1, n) + 1, \quad j_2 = \mathrm{mod}(3i - 1, n) + 1, \quad j_3 = \mathrm{mod}(5i - 1, n) + 1,$$

$$j_4 = \mathrm{mod}(7i - 1, n) + 1, \quad j_5 = \mathrm{mod}(11i - 1, n) + 1,$$

$$\bar{x}_i = 0.5, \quad i \ge 1.$$

Problem 53. SPARSQUR

$$F(x) = \frac{1}{8} \sum_{i=1}^{n} i \Big[x_i^2 + x_{j_1}^2 + x_{j_2}^2 + x_{j_3}^2 + x_{j_4}^2 + x_{j_5}^2 \Big]^2,$$

$$j_1 = \mathrm{mod}(2i - 1, n) + 1, \quad j_2 = \mathrm{mod}(3i - 1, n) + 1, \quad j_3 = \mathrm{mod}(5i - 1, n) + 1,$$

$$j_4 = \mathrm{mod}(7i - 1, n) + 1, \quad j_5 = \mathrm{mod}(11i - 1, n) + 1,$$

$$\bar{x}_i = 0.5, \quad i \ge 1.$$

Problem 54. SPMSRTLS

$$F(x) = \sum_{i=1}^{m} \sum_{k=1}^{5} f_{jk}(x),$$

$$\begin{aligned}
f_{jk}(x) &= (x_{j-4}x_{j-1} - p_{j-4}p_{j-1})^2, \quad \mathrm{mod}(k,5) = 1,\ i > 2, \\
f_{jk}(x) &= (x_{j-3}x_{j-1} + x_{j-1}x_j - p_{j-3}p_{j-1} - p_{j-1}p_j)^2, \quad \mathrm{mod}(k,5) = 2,\ i > 1, \\
f_{jk}(x) &= (x_j^2 - p_j^2)^2, \quad \mathrm{mod}(k,5) = 3, \\
&+ (x_{j-2}x_{j-1} - p_{j-2}p_{j-1})^2, \quad i > 1, \\
&+ (x_{j+2}x_{j+1} - p_{j+2}p_{j+1})^2, \quad i < m, \\
f_{jk}(x) &= (x_{j+3}x_{j+1} + x_{j+1}x_j - p_{j+3}p_{j+1} - p_{j+1}p_j)^2, \quad \mathrm{mod}(k,5) = 4,\ i < m, \\
f_{jk}(x) &= (x_{j+4}x_{j+1} - p_{j+4}p_{j+1})^2, \quad \mathrm{mod}(k,5) = 0,\ i < m-1,
\end{aligned}$$

$$\bar{x}_i = p_i/5, \quad p_i = \sin i^2, \quad j = 3(i-1)+1, \quad m = (n+2)/3.$$

Problem 55. SROSENBR

$$F(x) = \sum_{i=1}^{n/2} \Big[100(x_{2i} - x_{2i-1}^2)^2 + (x_{2i-1} - 1)^2 \Big],$$

$$\bar{x}_i = -1.2, \quad \mathrm{mod}(i,2) = 1, \quad \bar{x}_i = 1.0, \quad \mathrm{mod}(i,2) = 0.$$

Problem 56. TOINTGSS

$$F(x) = \sum_{i=1}^{n-2} \Big[\Big(\frac{10}{n+2} + x_{i+2}^2 \Big) \Big(2 - \exp\Big[- \frac{(x_i - x_{i+1})^2}{0.1 + x_{i+2}^2} \Big] \Big) \Big],$$

$$\bar{x}_i = 3.0, \quad i \geq 1.$$

Problem 57. TQUARTIC

$$F(x) = (x_1 - 1)^2 + \sum_{i=1}^{n-1} (x_1^2 - x_{i+1}^2)^2,$$

$$\bar{x}_i = 0.1, \quad i \geq 1.$$

Problem 58. WOODS

$$\begin{aligned}
F(x) &= \sum_{i=1}^{n/4} \Big[100(x_{4i-2} - x_{4i-3}^2)^2 + (1 - x_{4i-3})^2 + 90(x_{4i} - x_{4i-1}^2)^2 \\
&+ (1 - x_{4i-1})^2 + 10(x_{4i-2} + x_{4i} - 2)^2 + \frac{1}{10}(x_{4i-2} - x_{4i})^2 \Big],
\end{aligned}$$

$$\bar{x}_i = -3.0, \quad \mathrm{mod}(i,2) = 1, \quad \bar{x}_i = -1.0, \quad \mathrm{mod}(i,2) = 0.$$

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