An algebraic study of partial predicates

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September 5, 2013

Outline

- Describe a way to model partial predicates
- Define a logic which preserves exactness in partial contexts
- Some issues of abstract algebraic logic
- Provide a Gentzen calculus for the logic

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Partial Predicates

A partial predicate over a set X is a pair $\langle A, B\rangle$ such that $A, B \subseteq X$ and $A \cap B = \emptyset$. $\mathcal{D}(X) := \{\langle A, B\rangle : \langle A, B\rangle \text{ is a partial predicate over } X\}$.

Given $\langle A, B\rangle, \langle C, D\rangle \in \mathcal{D}(X)$ we let

$$\langle A, B\rangle \cap \langle C, D\rangle := \langle A \cap C, B \cup D\rangle$$

$$\langle A, B\rangle \cup \langle C, D\rangle := \langle A \cup C, B \cap D\rangle$$

$$\neg\langle A, B\rangle := \langle B, A\rangle$$

$$n := \langle\emptyset, \emptyset\rangle.$$

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Partial Predicates

How to describe fields of partial sets? An algebra $\boldsymbol{A} = \langle A, \wedge, \vee, \neg, n \rangle$ is a DMF lattice if it is a distributive lattice that satisfies the following equations:

$$x \vee y = \neg(\neg x \wedge \neg y) \qquad x \wedge y = \neg(\neg x \vee \neg y)$$

$$\neg\neg x = x \qquad \neg n = n$$

$$x \wedge \neg x \leq y \vee \neg y.$$

We will denote by DMF the variety of DMF lattices. Let $\mathbf{Z}_{2m+1}$ the $2m+1$-element DMF chain with universe $\{-m, \ldots, -1, n, 1, \ldots, m\}$.

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We are interested in logics related to the variety DMF. Let $\boldsymbol{A}$ be an algebra and $F \subseteq A$. $\theta \in \mathsf{Co}(\boldsymbol{A})$ is compatible with F if it satisfies the following condition

$$\text{if } a \in F \text{ and } \langle a, b \rangle \in \theta, \text{ then } b \in F.$$

The Leibniz congruence of F over $\boldsymbol{A}$ is the largest congruence over $\boldsymbol{A}$ compatible with F. We denote it by $\boldsymbol{\Omega} F$. The map $\boldsymbol{\Omega}\colon \mathcal{P}(A) \to \mathsf{Co}(\boldsymbol{A})$ such that

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Leibniz Hierarchy

Let $\boldsymbol{A}$ be an algebra. $F \subseteq A$ is a deductive filter of $\mathcal{L}$ if for every $\Gamma \cup \{\varphi\} \subseteq Fm$

$$\text{if } \Gamma \vdash_{\mathcal{L}} \varphi, \text{ then for every homomorphism } h \colon \boldsymbol{Fm} \to \boldsymbol{A}$$

$$\text{if } h[\Gamma] \subseteq F, \text{ then } h(\varphi) \in F.$$

We let $\mathcal{F}i_{\mathcal{L}}(\boldsymbol{A}) := \{F \subseteq A : F \text{ is a deductive filter of } \mathcal{L}\}$.

We let

$$\mathrm{Mod}^{*}\mathcal{L} = \{\langle \boldsymbol{A}, F\rangle : F \in \mathcal{F}i_{\mathcal{L}}(\boldsymbol{A}) \text{ and } \boldsymbol{\Omega} F = \Delta\}$$

$$\mathrm{Alg}^{*}\mathcal{L} = \{\boldsymbol{A} : \text{there is } F \in \mathcal{F}i_{\mathcal{L}}(\boldsymbol{A}) \text{ such that } \boldsymbol{\Omega} F = \Delta\}.$$

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A logic $\mathcal{L}$ is protoalgebraic if there is a set $\Delta(x, y, \vec{z}) \subseteq Fm$ such that for every algebra $\boldsymbol{A}$, $F \in \mathcal{F}i_{\mathcal{L}}(\boldsymbol{A})$ and $a, b \in A$

$$\langle a, b\rangle \in \boldsymbol{\Omega} F \iff \Delta^{\boldsymbol{A}}(a, b, \vec{c}) \subseteq F \text{ for every } \vec{c} \in A.$$

Theorem

A logic $\mathcal{L}$ is protoalgebraic if and only if $\boldsymbol{\Omega}\colon \mathcal{P}(A) \to \mathrm{Co}(\boldsymbol{A})$ is monotone over $\mathcal{F}i_{\mathcal{L}}(\boldsymbol{A})$ for every algebra $\boldsymbol{A}$.

A logic $\mathcal{L}$ is truth-equational if there is a set $E(x) \subseteq Eq$ such that $F = \{a \in A : \boldsymbol{A} \models E(a)\}$ for every algebra $\langle \boldsymbol{A}, F\rangle \in \mathrm{Mod}^{*}\mathcal{L}$.

Theorem ([3])

A logic $\mathcal{L}$ is truth-equational if and only if $\boldsymbol{\Omega}\colon \mathcal{P}(A) \to \mathrm{Co}(\boldsymbol{A})$ is completely order reflecting over $\mathcal{F}i_{\mathcal{L}}(\boldsymbol{A})$ for every algebra $\boldsymbol{A}$.

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Theorem

If $\mathbf{Z}_3 \in \mathrm{Alg}^*\mathcal{L}$, then $\mathcal{L}$ is not protoalgebraic.

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If $\mathbf{Z}_5 \in \mathrm{Alg}^*\mathcal{L}$, then $\mathcal{L}$ is not truth-equational.

Proof.

- Up to equivalence terms in one variable in DMF are $\mathcal{T}(x) := \{n, x \vee \neg x, x \vee n, \neg x, \neg x \vee n, x \wedge \neg x, x \wedge n, \neg x \wedge n, x, \neg x\}$;
- if $\alpha, \beta \in \mathcal{T}(x)$ and $\boldsymbol{A} \in$ DMF, then $\{a \in A : \boldsymbol{A} \models \alpha(a) \approx \beta(a)\} \in \{\{n\}, \downarrow n, \uparrow n, A\}$;
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A concrete logic $\mathcal{L}_{\{1\}}$:

$$\Gamma \vdash_{\mathcal{L}_{\{1\}}} \varphi \iff \text{ for every homomorphism } h\colon \boldsymbol{Fm} \to \mathbf{Z}_3$$

$$\text{if } h[\Gamma] \subseteq \{1\}, \text{ then } h(\varphi) = 1.$$

Observe that

(i) $\mathcal{L}_{\{1\}}$ is finitary;

(ii) $\mathcal{L}_{\{1\}}$ has no theorems;

(iii) $\mathcal{L}_{\{1\}}$ is neither protoalgebraic, nor truth-equational, nor selfextensional.

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Let $\boldsymbol{A} \in$ DMF. $\mathcal{F}i_n(\boldsymbol{A})$ is the set of lattice filters F such that $n \notin F$ and

$$\text{if } a \vee n \in F, \text{ then } a \in F.$$

Lemma

Let $\boldsymbol{A} \in$ DMF. $\mathcal{F}i_{\mathcal{L}_{\{1\}}}(\boldsymbol{A}) = \mathcal{F}i_n(\boldsymbol{A}) \cup \{\emptyset, A\}$.

Lemma ([1])

Let $\boldsymbol{A}$ be a De Morgan Lattice, $F \subseteq A$ a lattice filter and $a, b \in A$. $\langle a, b\rangle \in \boldsymbol{\Omega} F$ if and only if for every $c \in A$ the following conditions hold

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Theorem

Let $\boldsymbol{A}$ be non-trivial. $\langle \boldsymbol{A}, F \rangle \in \mathrm{Mod}^{\star}\mathcal{L}_{\{1\}}$ if and only if the following conditions hold:

(i) $\boldsymbol{A} \in \mathrm{DMF}$;

(ii) $\boldsymbol{A}$ *has a maximum* 1 *and* $F = \{1\}$;

(iii) *if* $a < b$*, then there is* $c \in A$ *such that* $a \vee c < b \vee c = 1$ *for every* $a, b \geq n$.

A logic $\mathcal{L}$ is uniequational if there is a set $E(x, \overline{y}) \subseteq Eq$ such that $F = \{a \in A : \boldsymbol{A} \models E(a, \overline{b}) \text{ for every } \overline{b} \in A\}$ for every $\langle \boldsymbol{A}, F \rangle \in \mathsf{Mod}^{*}\mathcal{L}$ such that $F \neq \emptyset$.

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Corollary

Let $\mathcal{L}$ be an uniequational logic. $\mathcal{L}$ is truth-equational if and only if it has theorems.

Lemma

$\mathcal{L}_{\{1\}}$ is uniequational with defining equations $\{x \wedge y \approx y\}$.

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Let $\mathcal{L}$ be an uniequational logic. $\mathcal{L}$ is truth-equational if and only if it has theorems.

Lemma

$\mathcal{L}_{\{1\}}$ is uniequational with defining equations $\{x \wedge y \approx y\}$.

Leibniz-Reduced Models

Theorem ([2])

$\boldsymbol{A} \in \mathrm{Alg}^{\star}\mathcal{CPC}_{\{\wedge,\vee,\bot\}}$ *if and only if the following conditions hold:*

(i) $\boldsymbol{A} \in \mathrm{DL}_{\bot}$;

(ii) $\boldsymbol{A}$ *has a maximum* 1;

(iii) *if* $a < b$, *then there is* $c \in A$ *such that* $a \vee c < b \vee c = 1$ *for every* $a, b \in A$.

Corollary

Let $\boldsymbol{A} \in \mathrm{DMF}$. $\boldsymbol{A} \in \mathrm{Alg}^{*}\mathcal{L}_{\{1\}}$ *if and only if* $\uparrow(\boldsymbol{A}) \in \mathrm{Alg}^{\star}\mathcal{CPC}_{\{\wedge,\vee,\bot\}}$.

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Let $\boldsymbol{A}$ be an algebra and $\mathcal{C} \subseteq \mathcal{P}(A)$ be a closure system. The Tarski congruence of $\mathcal{C}$ over $\boldsymbol{A}$ is the largest congruence over $\boldsymbol{A}$ compatible with every $X \in \mathcal{C}$. We denote it by $\widetilde{\boldsymbol{\Omega}}\mathcal{C}$. $\langle \boldsymbol{A}, \mathcal{C} \rangle$ is a g-model of $\mathcal{L}$ if $X \in \mathcal{F}i_{\mathcal{L}}(\boldsymbol{A})$ for every $X \in \mathcal{C}$.

$$\mathrm{Alg}\mathcal{L} = \{\boldsymbol{A} : \text{ there is a g-models } \langle \boldsymbol{A}, \mathcal{C} \rangle \text{ of } \mathcal{L} \text{ such that } \widetilde{\boldsymbol{\Omega}}\mathcal{C} = \Delta\}.$$

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A Gentzen System

Let $\mathfrak{G}$ be the Gentzen system (premisses are non-empty finite sets of formulas) defined by the following rules:

$$\alpha \rhd \alpha\,(R) \qquad \frac{\Gamma \rhd \alpha}{\Gamma, \beta \rhd \alpha}(W) \qquad \frac{\Gamma \rhd \alpha \quad \Gamma, \alpha \rhd \beta}{\Gamma \rhd \beta}(Cut)$$

$$\frac{\Gamma, \alpha, \beta \rhd \gamma}{\Gamma, \alpha \wedge \beta \rhd \gamma}(\wedge\rhd) \qquad \frac{\Gamma \rhd \alpha \quad \Gamma \rhd \beta}{\Gamma \rhd \alpha \wedge \beta}(\rhd\wedge)$$

$$\frac{\Gamma, \alpha \rhd \gamma \qquad \Gamma, \beta \rhd \gamma}{\Gamma, \alpha \vee \beta \rhd \gamma}(\vee\rhd) \qquad \frac{\Gamma \rhd \alpha}{\Gamma \rhd \alpha \vee \beta}, \frac{\Gamma \rhd \alpha}{\Gamma \rhd \beta \vee \alpha}(\rhd\vee)$$

$$\frac{\Gamma, \alpha \rhd \beta}{\Gamma \neg\neg\alpha, \rhd \beta}(\neg\rhd) \qquad \frac{\Gamma \rhd \alpha}{\Gamma \rhd \neg\neg\alpha}(\rhd\neg)$$

$$\frac{\Gamma, \neg\alpha \rhd \gamma \qquad \Gamma, \neg\beta \rhd \gamma}{\Gamma, \neg(\alpha \wedge \beta) \rhd \gamma}(\neg \wedge \rhd) \qquad \frac{\Gamma \rhd \neg\alpha}{\Gamma \rhd \neg(\alpha \wedge \beta)}, \frac{\Gamma \rhd \neg\beta}{\Gamma \rhd \neg(\alpha \wedge \beta)}(\rhd\neg\wedge)$$

$$\frac{\Gamma, \neg\alpha, \neg\beta \rhd \gamma}{\Gamma, \neg(\alpha \vee \beta) \rhd \gamma}(\neg \vee \rhd) \qquad \frac{\Gamma \rhd \neg\alpha \qquad \Gamma \rhd \neg\beta}{\Gamma \rhd \neg(\alpha \vee \beta)}(\rhd\neg\vee)$$

$$n \vee \neg n \rhd \alpha\,(n\rhd) \qquad \frac{\Gamma \rhd \alpha}{\Gamma, \neg\alpha \rhd n}(\rhd n)$$

Let $\tau \colon \mathcal{P}(Seq) \longrightarrow \mathcal{P}(Eq) \colon \rho$ be the residuated mappings defined as

$$\tau(\Gamma \rhd \alpha) = \bigwedge \Gamma \leq \alpha \vee n \qquad \rho(\alpha \approx \beta) = \{\alpha \lhd \rhd \beta, \neg\alpha \lhd \rhd \neg\beta\}$$

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Theorem

$\mathfrak{G}$ is algebraizable with equivalent algebraic semantics DMF *via τ and ρ.*

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Future Work

- Begin a general study of uniequational logics and their relation with algebraic semantics;
- Develop the relation between uniequational logics and the Frege hierarchy;
- There are interesting examples of logics which arise from an analogous weakening of protoalgebraic logics?
- Is there a logic which has a universal semantics and is neither uniequational, nor has an algebraic semantics?

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