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Games, equilibrium semantics and many-valued connectives

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Motivation:

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(1) Nash equilibria for languages of imperfect information

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Aim of the talk:

to show that the two approaches nicely augment each other and fit into a common frame that opens new perspectives for both: incomplete information as well as many-valued connectives.

Plan of the talk

- very brief reminder on equilibrium semantics
- brief reminder on Giles's game for Łukasiewicz logic
- Hintikka-Sandu games as dispersive experiments
- independence-friendly Łukasiewicz logic?
- more connectives from incomplete information
- summary, perspectives

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- Hintikka-Sandu games as dispersive experiments
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The main message in three lines:

Imperfect information in semantic games can explain intermediate truth values, but also gives raise to a richer set of connectives and quantifiers. However, Giles's more general notion of a state is used.

The classic semantic game (Hintikka's game)

Proponent **P** defends/asserts and Opponent **O** attacks the claim that a formula F is true under a fixed interpretation (model) $\mathcal{I}$.

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Proponent **P** defends/asserts and Opponent **O** attacks the claim that a formula F is true under a fixed interpretation (model) $\mathcal{I}$.

Rules of the game:

P asserts $F \wedge G$: **O** picks F or G, **P** asserts F or G, accordingly

P asserts $F \vee G$: **P** asserts F or G, according to her own choice

P asserts $\neg F$: **P** asserts F, but the roles (**P**/**O**) are switched

P asserts $\forall x F(x)$: **O** picks $a \in |\mathcal{I}|$ and **P** asserts $F(a)$

P asserts $\exists x F(x)$: **P** picks $a \in |\mathcal{I}|$ and **P** asserts $F(a)$

Winning condition:
P (after switch: **O**) wins if an atom that is true in $\mathcal{I}$ is reached

Central Fact: (characterization of Tarski's "truth in a model")

P has a winning strategy iff F is true in $\mathcal{I}$

Imperfect information (Hintikka-Sandu game)

The players may not know the full history of a game run.

This triggers a richer syntax (**IF logic**):

E.g., $\forall x (\exists y/\{x\}) x = y$ means that **P** has to pick the witness for y without knowing which element in $|\mathcal{I}|$ was picked by **O** for x.

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Important properties:

- determinedness is lost: e.g., neither **P** nor **O** has a winning strategy for $\forall x (\exists y / \{x\}) x = y$ if there is more than one element in the domain $|\mathcal{I}|$
- IF logic is more expressive: the set of formulas for which **P** has a winning strategy corresponds to valid formulas of existential second order logic
- IF logic is non-classical: E.g., $A \vee \neg A$ is not valid, but
- except for "slashing" the syntax remains with $\vee, \wedge, \neg, \forall, \exists$

Equilibrium Semantics

In the classical Hintkka game backward induction yields the value of a game for F with respect to $\mathcal{I}$:
$\|F\|_{\mathcal{I}} = 1$ ... **P** has a winning strategy for F w.r.t. $\mathcal{I}$
$\|F\|_{\mathcal{I}} = 0$ ... **O** has a winning strategy for F w.r.t. $\mathcal{I}$

For general IF formulas one still obtains a unique Nash equilibrium for mixed strategies as value:
E.g. the value of $\forall x \big(\exists y/\{x\}\big) x = y$ ("matching pennies") is $1/n$, where n is the cardinality of $\mathcal{I}$. Similarly $\forall x \big(\exists y/\{x\}\big) x \neq y$ ("inverse matching pennies") has value $(n-1)/n$.

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Equilibrium semantics leads to truth functional semantics for the "weak fragment" of Łukasiewicz logic:
$\|\neg F\|_{\mathcal{I}} = 1 - \|F\|_{\mathcal{I}}$
$\|F \vee G\|_{\mathcal{I}} = \max(\|F\|_{\mathcal{I}}, \|G\|_{\mathcal{I}})$ (analogously for $\exists$)
$\|F \wedge G\|_{\mathcal{I}} = \min(\|F\|_{\mathcal{I}}, \|G\|_{\mathcal{I}})$ (analogously for $\forall$)

Every rational $\in [0,1]$ is a value of some F in some finite $\mathcal{I}$

Giles's analysis of approximate reasoning

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Meaning of connectives specified by dialogue rules (Lorenzen):

Let **X**/**Y** stand for **P**/**O** or for **O**/**P**

X asserts	'attack' by **Y**	answer by **X**
$A \to B$	A	B
$A \vee B$	'?'	A or B (**X** chooses)
$A \wedge B$	'l?' or 'r?' (**Y** chooses)	A or B (accordingly)
$A \,\&\, B$	'?'	A and B

Note: $\neg A$ abbreviates $A \to \bot$

The answer $\bot$ ('I loose') is allows allowed

(= Giles's *"principle of limited liability"* – only relevant for &)

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Still missing:

- winning conditions for atomic states
- regulations defining admissible runs of a game

ad: winning conditions

Giles's idea: Players bet on the truth of their (atomic) claims!
(Yes/no-)experiments — that may be dispersive — decide.

- **P** pays 1€ to **O** for each false atomic assertions made by him, **O** pays 1€ to **P** for each false atomic assertion made by her

A final states $[p_1, \ldots, p_m \,\|\, q_1, \ldots, q_n]$ results in a pay-off of

$$\Big(\sum_{i=1}^{m} \langle p_i \rangle - \sum_{j=1}^{n} \langle q_j \rangle\Big)€ \quad \text{for me}$$

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ad: regulations

Constraints on dialogues like the following suffice:

$(R_{\rightarrow})$ If **O** attacks **P**'s assertion of $A \rightarrow B$ by claiming A, then, in reply, **P** has to assert also B eventually.

Attacked formulas are removed from the current state.
No particular regulation for the order of moves is required!

Definition:

A game for F w.r.t. $\mathcal{I}$ has (risk-)value x if **P** has a strategy to limit his loss to x€, while **O** has a strategy to guarantee a win of x€.

Giles's Theorem:

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Remarks:

- standard rules for $\forall$ and $\exists$ work under some provisions: consider 'limit values' or just witnessed models
- the game can be generalized in different ways to cover various other many-valued logics
- connection to proof systems: analytic (hypersequent) proofs arise from systematic search for winning strategies

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Is there any non-trivial common ground at all?

HS-games as dispersive experiments

Idea:

Analyze each atomic assertion in a G-game as initial assertion of an HS-game. In other words: consider every run of an HS-game as dispersive experiment.

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A two-tiered language:

$IF := \mathrm{atom} | \neg IF | IF \vee IF | IF \wedge IF | \forall v / \{v_1, ..., v_n\} IF | \forall v / \{v_1, ..., v_n\} IF$

$ŁF := IF | \bot | ŁF \vee' ŁF | ŁF \wedge' ŁF | ŁF \to ŁF | ŁF \,\&\, ŁF \; \left[| \forall v ŁF | \exists v ŁF \right]$

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Game semantics:

(1) play the G-game to reduce *ŁF*-formulas to *IF*-formulas

(2) play an independent HS-game for each *IF*-formula

(3) evaluate like in G-games: pay 1€ for each lost HS-(sub)game

Note: risk-values are sums of inverted equilibrium values.

Definition: (truth) value = inverted risk-value

HS-games as dispersive experiments (ctd.)

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Some simple examples:

Let $MP = \forall x\big(\exists y/\{x\}\big)x = y$ and $IMP = \forall x\big(\exists y/\{x\}\big)x \neq y$ and let n be the cardinality of the model $\mathcal{I}$

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- *MP*: **P** has to pay 1€ with probability $(n-1)/n$
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Main idea of randomized choices for (semi-fuzzy) quantifiers: instead of letting **P** or **O** pick the witnessing constant, consider random witnesses (w.r.t. uniform distribution over the domain).

This turns out to match various 'vague' (semi-fuzzy) quantifiers.

E.g., '**Many** $x\,F(x)$' might be modeled as '*A randomly picked domain element satisfies F with probability* $\geq \gamma$' (some threshold)

The basic random choice quantifier $\sqcap$ is given by the rule:

P asserts $\sqcap x\, F(x)$: **P** asserts $F(a)$ for a randomly picked $a \in |\mathcal{I}|$

The basic random choice quantifier Π is given by the rule:

P asserts $\Pi x\, F(x)$: **P** asserts $F(a)$ for a randomly picked $a \in |\mathcal{I}|$

NB: Many more interesting quantifiers can be defined similarly.
E.g. proportionality quantifiers modeling **about half**, **few**, **many**.
These can be reduced to Π within Giles's game!
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Answer:
$\Pi x\, F(x) \approx \forall x/\{x, \ldots\} F(x) \Leftrightarrow \exists x/\{x, \ldots\} F(x)$
In other words:
Picking x without any information amounts to randomized choice!

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Remark for experts on Łukasiewicz logic:
'**or**' could also be strong disjunction, leading also to value 1.
It can also be modeled in Giles's game and is truth functional!

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- many more variants of connectives and quantifiers arise
- some similarity with game semantics for linear logic,
  but even more with Japaridze's Computability Logic

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- 'commonsense conjunction': To win F **and** G
  P has to win both: a game for F and a game for G
- many more variants of connectives and quantifiers arise
- some similarity with game semantics for linear logic,
  but even more with Japaridze's Computability Logic

Message: It's fine to stick just with Hintikka's rules for $\vee$, $\wedge$, $\neg$ in classical logic; but incomplete information widens the playground and naturally leads to further (variants of) connectives!

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  - more connectives arising in the incomplete information scenario
- overall, we obtain a rich new field of investigation!