The importance of Structure in Algebraic Preconditioners

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Outline

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Solving large, sparse SPD systems by iterative methods

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- providing just sufficient approximation of the algebraic problem if this makes computations faster
- Our target is robustness, not a fragile power

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- Basic ways of improvements
- Experimental results showing some structure-based effects

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- starting with first order factorizations $N = M - A = O(h)$

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Incomplete decompositions

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- Crucial moment: paper by Meijerink and van der Vorst (1977) recognizing the potential of incomplete decompositions for preconditioning.
- Incomplete decompositions classified by adding (ℓ) after the name. Starting to denote them by number of additional diagonals in simple problems $\rightarrow IC(\ell)$.

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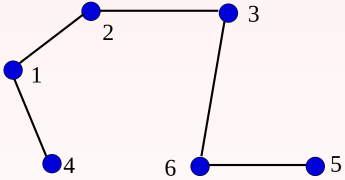

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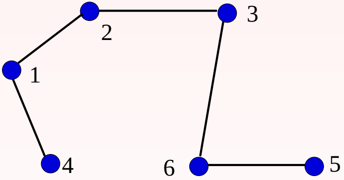

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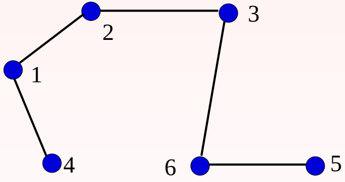

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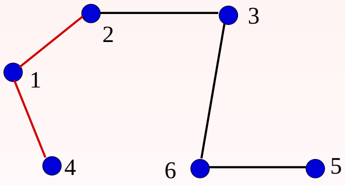

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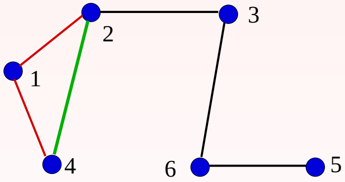

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- Practically: A fill entry is permitted provided $level(i,j) \le \ell$.

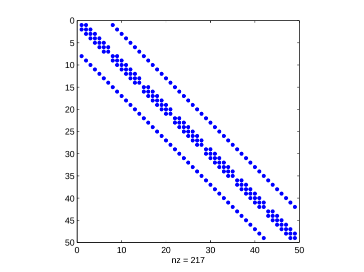
$$level(i,j) = \min_{1 \le l \le \min\{i,j\}} \{level(i,l) + level(l,j) + 1\}$$

(one of more slightly different definitions)

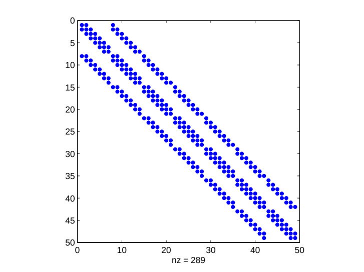
Example: Structure-based preconditioners

Incomplete decompositions

General patterns: an example



IC(0)



IC(1)

Incomplete decompositions

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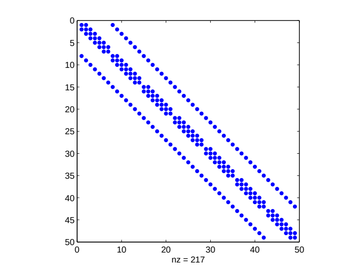

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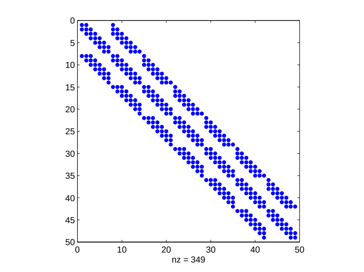

IC(2)

Incomplete decompositions

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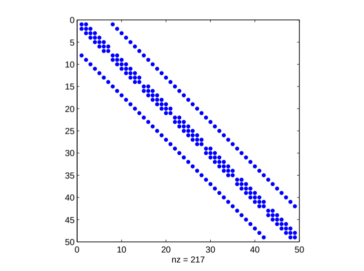

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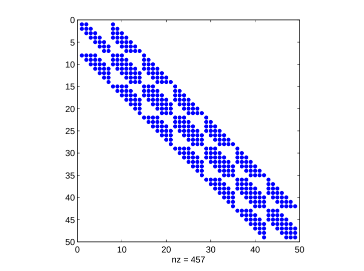

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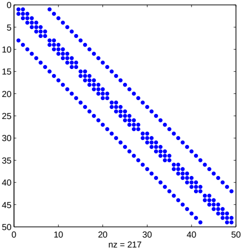

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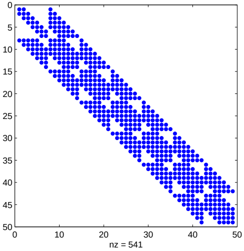

IC(4)

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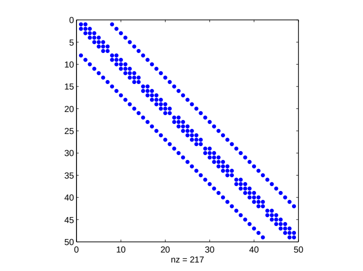

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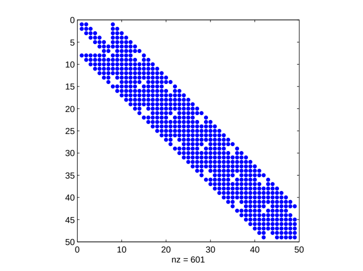

IC(5)

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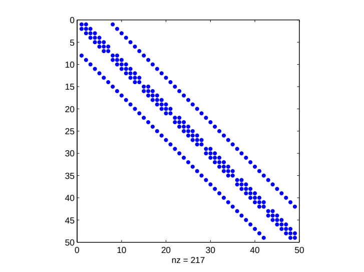

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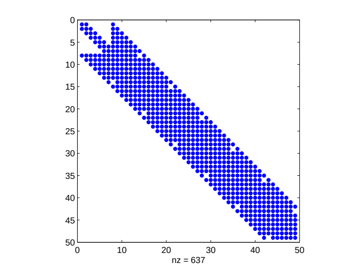

IC(6)

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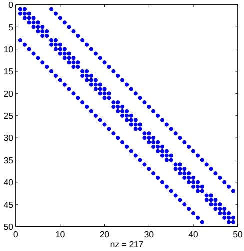

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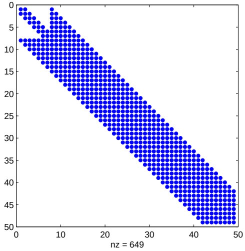

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- But: Decay of entries away from diagonal may help a lot.

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- The importance of error matrix $E = A - LL^T$ understood (Duff, Meurant, (1989)) and exploited (D'Azevedo, Forsyth, Tang, 1992)

- First simple combination of level-based approaches with dropping: D'Azevedo, Forsyth, Tang, (1992a).

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- Generally, convergence behavior can be far from predictable
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- Our MI22 preconditioner is a new way to use level-based information, memory prediction and dropping at the same time.

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First component of our approach: new setting of levels

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- Each potential fill entry l_{ij} is assigned a level

$$level(i,j) = \min_{1 \leq l \leq \min\{i,j\}} \{level(i,l) + level(l,j) + 1\}.$$

A fill entry is permitted provided $level(i,j) \leq k$.

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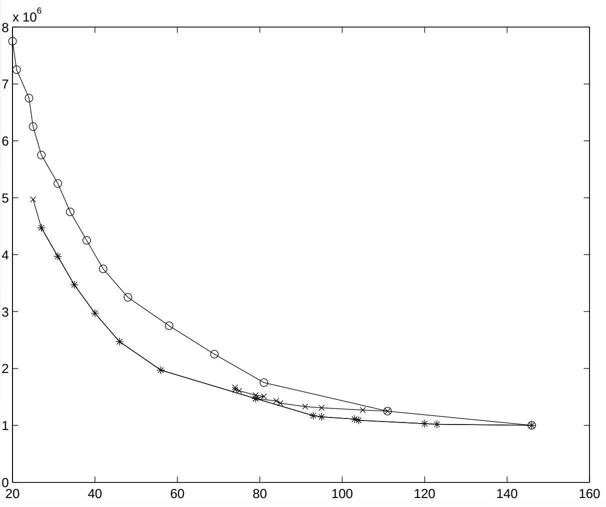

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- Open problem: determine more sophisticated rules to preassign levels.

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- Nonuniform distribution based on the elimination tree and its row subtrees

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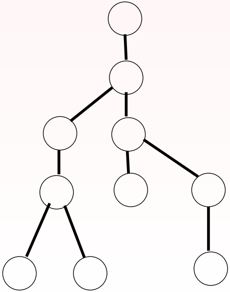

- Nonuniform distribution based on the elimination tree and its row subtrees

Second component of our approach: keeping structure

Integrate the predefined factor structure with dropping

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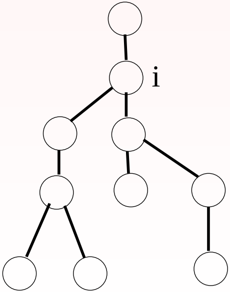

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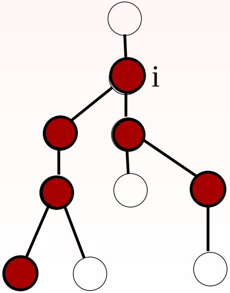
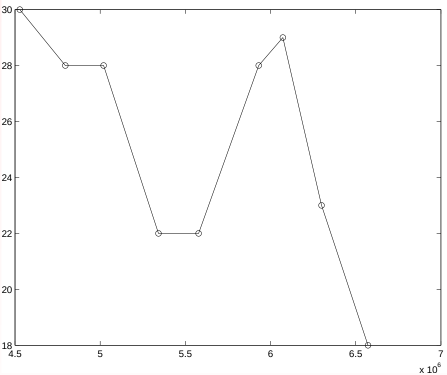
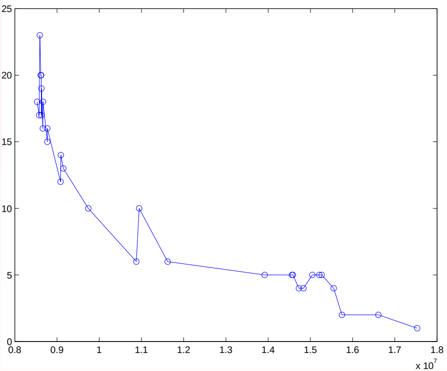


Second component of our approach: keeping structure

Experiments with memory multiplier $\theta = 1$: more difficult problem



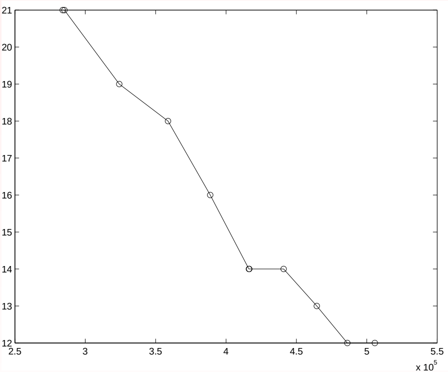


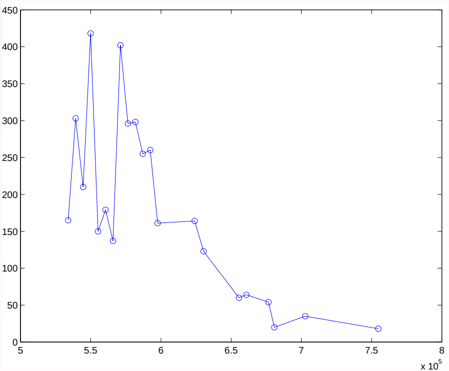
MI22 ILUT (ICT)

NASASRB, structural mechanics, n=54870

Second component of our approach: keeping structure

Experiments with memory multiplier $\theta = 1$: simpler problem





MI22 ILUT (ICT)

S1RMT3M1, cylindrical shell problem, n=5489

MI22 with levels versus $IC(\tau)$(also via MI22)

TUBE1, cylindrical shell, n=21498

struc	drop=0.0		drop=10^{-7}	
level	size	its	size	its
5	1250952	†	1227570	†
6	1660827	429	1618808	423
7	1807337	405	1756733	408
8	2178312	272	2104496	281
9	2368289	260	2280081	267
10	3026431	184	2873613	185
11	3968731	426	3656826	335
12	4874629	†	4398086	†
13	5849563	†	5178688	†
14	6840871	664	5938543	647
15	7838623	262	6680235	215

IC(τ)	size	its
55	280626	†
50	1458024	†
45	2076970	†
40	2252687	†
1e-3	16139618	†
1e-4	9001342	†
5e-5	9649083	471
2e-5	9610841	87
1e-5	10050227	18
5e-6	10741254	6
1e-6	12451396	2
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But: Reorderings may minimize the effect.

MI22 and memory multiplier $\theta < 1$

simple problem, 2D POISSON on a square, n=10000

memory	drop=0.0		drop=10^{-4}	
0.2	10000	160	10000	160
0.3	11880	226	11880	226
0.4	15840	205	15840	205
0.5	19800	155	19800	155
0.6	27729	141	27721	142
0.7	35597	111	35524	111
0.8	39583	63	39583	63
0.9	39584	58	39584	63
1	39601	41	39601	41
1.5	59401	42	59401	43
2	79202	42	79202	42
3	118803	42	118803	40
4	158404	42	158404	42
5	198005	44	198005	39
8	316808	42	316808	32
10	396010	42	396010	22
15	594015	37	471092	6

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