On the Way to Robust Algebraic Preconditioners

Miroslav Tůma

Institute of Computer Science,
Academy of Sciences of the Czech Republic,
Pod Vodárenskou věží 2, 18207 Praha 8, Czech Republic

e-mail: `tebbens@cs.cas.cz, tuma@cs.cas.cz`

Despite the enormous progress in improving efficiency of algorithms based on Gaussian elimination for solving large sparse systems of linear equations, there is still a strong need for analyzing and improvements of the algorithms, in particular, in the field of incomplete decompositions.

We consider the construction of robust preconditioners for iterative methods. In particular, we are interested in algebraic procedures for incomplete factorizations. The preconditioners are used for the solution of the system

$$Ax = b$$

for right-hand side vector $b \in \mathbb{R}^n$, vector $x \in \mathbb{R}^n$ of unknowns, and large sparse system matrix $A \in \mathbb{R}^{n \times n}$. We would like to get robust preconditioners rather by *new formulation* of the algorithmic scheme of the factorization than by the means of matrix preprocessings or ad hoc modifications of matrix factors.

One recent group of incomplete factorizations can be linked to factorized approximate inverses. For example, information from the inverse factors forms a crucial part of condition estimators and such procedures may be successfully used for dropping control in the incomplete factorization [1], [2]. Another step in this direction is represented by the computation of breakdown-free incomplete factorization RIF [3] via an intermediate factorized approximate inverse.

In this talk we will describe the new algorithm which computes both direct and inverse incomplete factors of A at the same time. The order of computation of intermediate quantities in this algorithm may enable to monitor conditioning of these factors, and, in addition, the algorithm can be combined with various dropping throughout the incomplete factorization. To get a flavour of the result, consider here, the simplest case of this factorization of the symmetric and positive definite A. Let $A = LDL^T$ be the factorization, where L is unit lower triangular and D is diagonal. The algorithm then computes the factors L, D, as well as the inverse of L. The basic overview of the result can be found in [5]. This result was originally derived using the theory for Sherman-Morrison-based approximate inverses [4] but it can be motivated in a different way as well.

In our talk we would like not only to describe our results for SPD matrices, but we will cover also its extension to the nonsymmetric case [6].

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