On efficient numerical approximation of the scattering amplitude

P. Tichý Czech Academy of Sciences, Czech Republic
tichy@cs.cas.cz

This talk presents results on efficient and numerically well-behaved estimation of the scalar value c^*x, where c^* denotes the conjugate transpose of c and x solves the linear system $Ax = b$, $A \in \mathbb{C}^{N\times N}$ is a nonsingular complex matrix and b and c are complex vectors of length N. In other words, we wish to estimate the *scattering amplitude* $c^*A^{-1}b$.

In our understanding, various approaches for numerical approximation of the scattering amplitude can be viewed as applications of the general mathematical concept of *matching moments model reduction*, formulated and used in applied mathematics by Vorobyev in his remarkable book [3]. Using the Vorobyev moment problem, matching moments properties of Krylov subspace methods can be described in a very natural and straightforward way, see [1]. This talk further develops the ideas from [1] into efficient estimates of $c^*A^{-1}b$, see [2].

We briefly outline the matching moment property of the Lanczos and Arnoldi algorithms, and specify techniques for estimating $c^*A^{-1}b$ with A non-Hermitian, including a new algorithm based on the BiCG method. We show its mathematical equivalence to the existing estimates which use a complex generalization of Gauss quadrature, and discuss its numerical properties. The proposed estimate will be compared with existing approaches using analytic arguments and numerical experiments on a practically important problem that arises from the computation of diffraction of light on media with periodic structure.

[1] Z. Strakoš, Model reduction using the Vorobyev moment problem, Numer. Algor., Vol. 51, pp. 363–379, July, 2009.
[2] Z. Strakoš and P. Tichý, On efficient numerical approximation of the scattering amplitude $c^*A^{-1}b$ via matching moments, submitted, 2009.
[3] Y. V. Vorobyev, Methods of moments in applied mathematics, Gordon and Breach Science Publishers, New York, 1965.

Joint work with Z. Strakoš (Czech Academy of Sciences, Czech Republic)