Numerical behavior of inexact saddle point solvers

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For large–scale saddle point problems, the application of exact iterative schemes and preconditioners may be computationally expensive. In practical situations, only approximations to the inverses of the diagonal block or the related cross-product matrices are considered, giving rise to inexact versions of various solvers. Therefore, the approximation effects must be carefully studied. In this talk we study numerical behavior of several iterative Krylov subspace solvers applied to the solution of large-scale saddle point problems. Two main representatives of the segregated solution approach are analyzed: the Schur complement reduction method, based on an (iterative) elimination of primary variables and the null-space projection method which relies on a basis for the null-space for the constraints. We concentrate on the question what is the best accuracy we can get from inexact schemes solving either Schur complement system or the null-space projected system when implemented in finite precision arithmetic. The fact that the inner solution tolerance strongly influences the accuracy of computed iterates is known and was studied in several contexts.

In particular, for several mathematically equivalent implementations we study the influence of inexact solving the inner systems and estimate their maximum attainable accuracy. When considering the outer iteration process our rounding error analysis leads to results similar to ones which can be obtained assuming exact arithmetic. The situation is different when we look at the residuals in the original saddle point system. We can show that some implementations lead ultimately to residuals on the the roundoff unit level independently of the fact that the inner systems were solved inexactly on a much higher level than their level of limiting accuracy. Indeed, our results confirm that the generic and actually the cheapest implementations deliver the approximate solutions which satisfy either the second or the first block equation to the working accuracy. In addition, the schemes with a corrected direct substitution are also very attractive. We give a theoretical explanation for the behavior which was probably observed or it is already tacitly known. The implementations that we pointed out as optimal are actually those which are widely used and suggested in applications.

References

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