On solvability of total least squares problems

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Let A be a real m by n matrix, and b a real m-vector. Consider estimating x from an orthogonally invariant linear approximation problem

$$Ax \approx b, \tag{1}$$

where the data b, A contain redundant and/or irrelevant information.

In *total least squares* (TLS) this problem is solved by constructing a minimal correction to the vector b and the matrix A such that the corrected system is compatible. Contrary to the standard least squares approximation problem, a solution of a TLS problem does not always exist. In addition, the data b, A can suffer from multiplicities and in this case a TLS solution may not be unique.

Classical analysis of TLS problems is based on the so called Golub - Van Loan condition $\sigma_{min}(A) > \sigma_{min}([b, A])$, see [2, 4]. This condition is, however, intricate through the fact that it is only sufficient but not necessary for the existence of a TLS solution.

A new contribution to the theory and computation of linear approximation problems was published in a sequence of papers [5, 6, 7], see also [3]. Here it is proved that the partial upper bidiagonalization [1] of the extended matrix $[b, A]$ determines a core approximation problem $A_{11}x_1 \approx b_1$, with the necessary and sufficient information for solving the original problem given by b_1 and A_{11}. The transformed data b_1 and A_{11} can be computed either directly, using Householder orthogonal transformations, or iteratively, using the Golub-Kahan bidiagonalization. It is shown how the core problem can be used in a simple and efficient way for solving the total least squares formulation of the original approximation problem.

In this contribution we discuss the necessary and sufficient condition for the existence of a TLS solution based on the core reduction, and mention work on extensions of the results to linear approximation problems with multiple right hand sides [8].

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