On solvability of total least squares problems

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Consider a linear approximation problem $A\,x \approx b$, where A is a real m by n matrix and b is a real m-vector. In *total least squares* (TLS) this problem is solved by constructing a minimal correction to the vector b and the matrix A such that the corrected system is compatible.

Contrary to the standard least squares approximation problem, a solution of a TLS problem does not always exist. In addition, the data b, A can suffer from multiplicities and in this case a TLS solution may not be unique. Classical analysis of TLS problems is based on the so called Golub - Van Loan condition $\sigma_{min}(A) > \sigma_{min}([b,\, A])$, see [1, 2]. This condition is, however, intricate through the fact that it is only sufficient but not necessary for the existence of a TLS solution.

A new contribution to the theory and computation of linear approximation problems was published in [3]. Here it is shown how the necessary and sufficient information for solving the approximation problem can be separated from the redundancies using the so called *core reduction*.

In this contribution we discuss the necessary and sufficient condition for the existence of a TLS solution based on the core reduction, and mention work on extensions of the results to problems with multiple right hand sides [4].

References

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