KRYLOV SUBSPACE RECYCLING FOR FAMILIES OF SHIFTED LINEAR SYSTEMS

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Abstract

We address the solution of a sequence of families of linear systems. For each family, there is a base coefficient matrix A_i, and the coefficient matrices for all systems in the family differ from A_i by a multiple of the identity, e.g.,

$$A_i x_i = b_i \quad \text{and} \quad (A_i + \sigma_i^{(\ell)} I) x_i^{(\ell)} = b_i \quad \text{for} \quad \ell = 1 \dots L_i,$$

where L_i is the number of shifts at step i. This is an important problem arising in various applications. We extend the method of subspace recycling to solve this problem by introducing a GMRES with subspace recycling scheme for families of shifted systems. This new method solves the base system using GMRES with subspace recycling while constructing approximate corrections to the solutions of the shifted systems at each cycle. These corrections improve the solutions of the shifted system at little additional cost. At convergence of the base system solution, GMRES with subspace recycling is applied to further improve the solutions of the shifted systems to tolerance. We present analysis of this method and numerical results involving systems arising in lattice quantum chromodynamics.