STABILITY ANALYSIS OF THE SPLIT BREGMAN ALGORITHM FOR DETERMINING OPTIMAL LAGRANGE PARAMETERS

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Abstract

The split Bregman algorithm for solving the ill-posed least squares problem $\|Ax - b\|^2$ subject to regularization condition $J(x)$, where J is typically an approximation for $\|Lx\|_{TV}$, has received significant attention dur to the work of Goldstein and Osher in 2008. They showed that the split Bregman algorithm provides an efficient approach for obtaining solutions of the regularized problem which is reformulated as $\|Ax - b\|^2 + \lambda\|Lx - d\|^2 + \mu J(d)$ where L is an appropriate operator and parameters λ, μ are regularization parameters, and can be solved by alternating updates over x and d. The former uses standard Tikhonov least squares problems for x and the latter uses a thresholding obtained via solution of a problem of the kind $\|d - c\|_2^2 + \mu J(d)$, for an updated vector c dependent on the x. Although the algorithm has received significant attention, the generation of optimal parameters λ and μ has not been addressed. We show by a suitable reformulation of the algorithm that the optimal λ is independent of the iteration step, and should be chosen as optimal for the Tikhonov problem. This result moreover demonstrates that the optimal value at each step is indeed step independent. This reduces the major question to determination of μ for which our results confirm that it is the ratio of λ to μ which is of most significance and determines the level of the threshold dependent on the level of the noise in the data. Numerical results will also be reported.