THE RICCATI METHOD FOR SINGULAR SUBSPACES OF LARGE SPARSE MATRICES

Gabriel Okša
Institute for Mathematics, Dept. of Informatics, Bratislava, Slovak Republic
e-mail: `Gabriel.Oksa@savba.sk`

Abstract

In the case of clusters of singular values, it is sometimes easier to estimate the left and right singular subspaces than the individual singular triplets. Moreover, after estimating the singular subspaces, the singular triplets can be obtained at small additional cost.

Let A be a given (sparse) matrix A of size $m \times n$. Our task is to find orthonormal (ON) bases $\hat{X} \in \mathbb{R}^{m \times k}$ and $\hat{Y} \in \mathbb{R}^{n \times k}$ of the left and right singular subspace, respectively, corresponding to k largest singular values of A. Starting with random initial matrices with k orthonormal (ON) columns $X \in \mathbb{R}^{m \times k}$ and $Y \in \mathbb{R}^{n \times k}$, let $(X|Z)$ and $(Y|W)$ be unitary. Then the required ON bases $\hat{X}$ and $\hat{Y}$ are given by $\hat{X} = X + ZP$, $\hat{Y} = X + W\tilde{P}$ where the *corrections* P and $\tilde{P}$ satisfy the Riccati algebraic system of two matrix equations. We show how this system is derived and discuss its solution using an orthogonal projection on the direct sum Krylov subspace $\mathcal{K}^{\ell}\left(\begin{bmatrix}0 & A\ A^{\star} & 0\end{bmatrix}, X \oplus Y\right)$ with the growing block dimension ℓ, $1 \leq \ell \leq m$. A fixed parameter m defines the maximum dimension mk of the updated generalized Rayleigh quotient; after reaching it, the block Arnoldi algorithm is restarted with the estimates closest to k largest singular values. The method can be easily adapted to the computations connected with k smallest singular values.

We present first numerical results computed in MATLAB using some sparse matrices from the Matrix Market Collection. It turns out that in the case of tight clusters of singular values our method is more robust than the MATLAB function `svds`.

Keywords: Riccati's correction equations, block Arnoldi algorithm, orthogonal projection, direct sum Krylov subspace

References

[1] Brandts, J.: *The Riccati algorithm for eigenvalues and invariant subspaces of matrices with inexpensive action.* Lin. Alg. Appl., 358 (2003), 335–365.

Acknowledgement: This research was supported by the VEGA grant no. 2/0003/11.