# A PRIORI ERROR ESTIMATES FOR NONLINEAR CONVECTIVE PROBLEMS

**Václav Kučera**
*Charles University in Prague, Faculty of Mathematics and Physics*
*Prague, Czech Republic*
*e-mail:* vaclav.kucera@email.cz

## Abstract

Standard textbook techniques for deriving a priori error estimates are well suited for problems like the heat equation, which possess a 'nice' structure (e.g. ellipticity, monotonicity). These parabolic techniques however fail for equations lacking such a structure, e.g. convective problems. Usually, one treats the convection-diffusion problem, and dominates the convective terms by the diffusion, which leads to estimates that blow up with respect to the diffusion coefficient going to zero and are not valid in the purely convective case. We shall present new error estimates for the purely convective and singularly perturbed cases, which are derived essentially using the parabolic technique. We build on estimates by Zhang and Shu (2004), which were limited to explicit schemes only. We extend their results to the method of lines using continuous mathematical induction and a nonlinear Gronwall lemma. For an implicit scheme, we show that the desired estimates cannot be obtained by standard arguments. To circumvent this obstacle, we construct a suitable continuation of the discrete solution with respect to time, so that we can again apply continuous mathematical induction. The key estimates can be applied to standard finite elements, as well as the discontinuous Galerkin method.