ON SPECIAL GRID TRANSFER OPERATORS FOR MULTIGRID METHODS

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Abstract

Based on the theory for Toeplitz matrices we discuss classical sufficient conditions to be satisfied from the grid transfer operators in order to obtain optimal two-grid and V-cycle multigrid methods. Based on this we derive relaxed conditions that allow for the construction of special grid transfer operators that are computationally less expensive while preserving optimality. The new conditions also allow to use rank deficient grid transfer operators, in this case the use of an intermediate iteration as a pre-smoother that is lacking the smoothing property is proposed.

Connected to the use of high-order polynomials as generating symbols for the system matrix and/or the grid transfer operators is the problem that the Toeplitz structure is destroyed on the coarser levels. We discuss some effective and computational cheap coarsening strategies found in the literature. For the case of Toeplitz matrices with a zero of order two (like the Laplacian) we prove the optimality of the V-cycle for these strategies, while for the high-order operators considered before we present numerical results showing near-optimal behavior while keeping the Toeplitz structure on the coarser levels.